

# Quantum control of spin correlations in ultracold lattice gases

P.Hauke, R.J.S., M.W.Mitchell & M.Lewenstein,  
PRA 87, 021601 (2013).



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M.W.Mitchell



# Quantum Polarization Spectroscopy

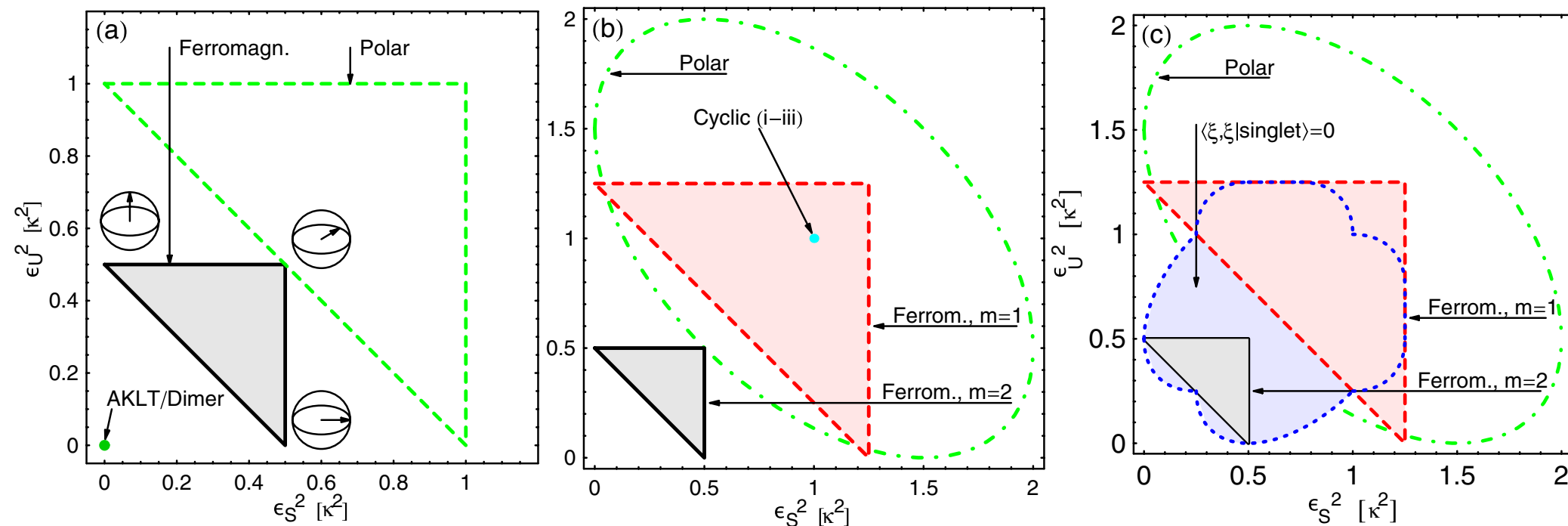
PRL **98**, 100404 (2007)

PHYSICAL REVIEW LETTERS

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## Quantum Polarization Spectroscopy of Ultracold Spinor Gases

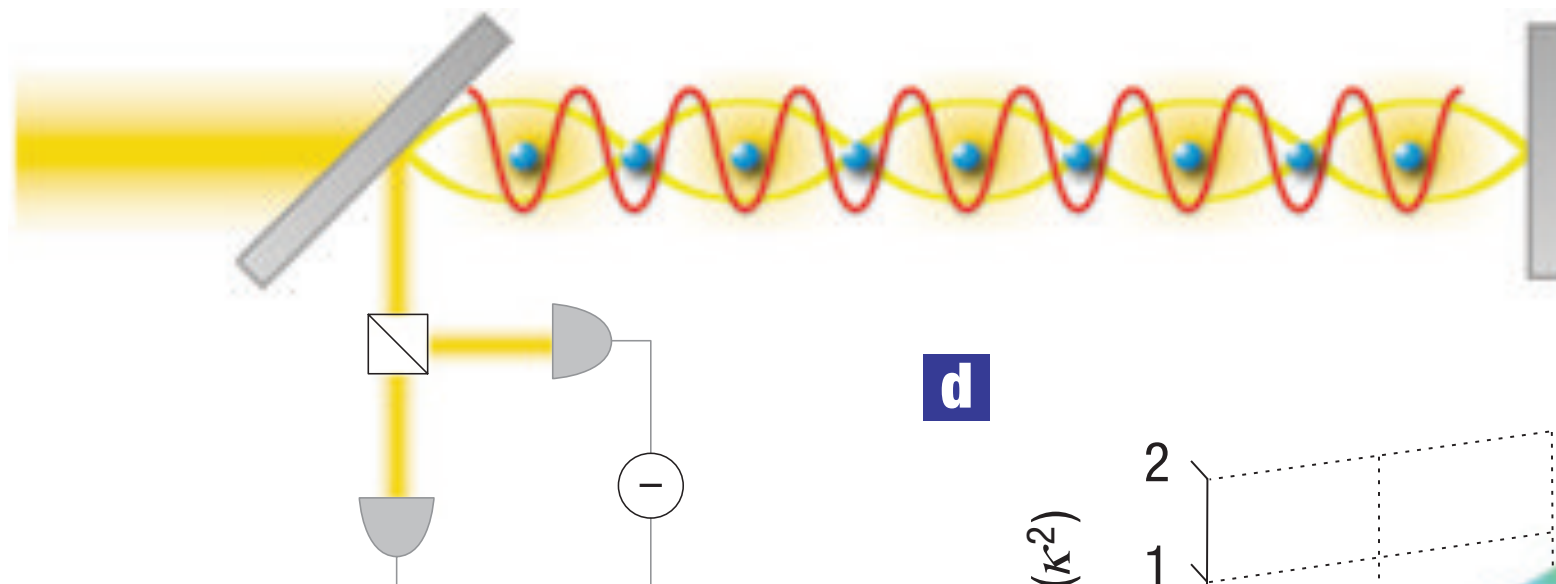
K. Eckert,<sup>1</sup> Ł. Zawitkowski,<sup>2</sup> A. Sanpera,<sup>3</sup> M. Lewenstein,<sup>4</sup> and E. S. Polzik<sup>5,6</sup>



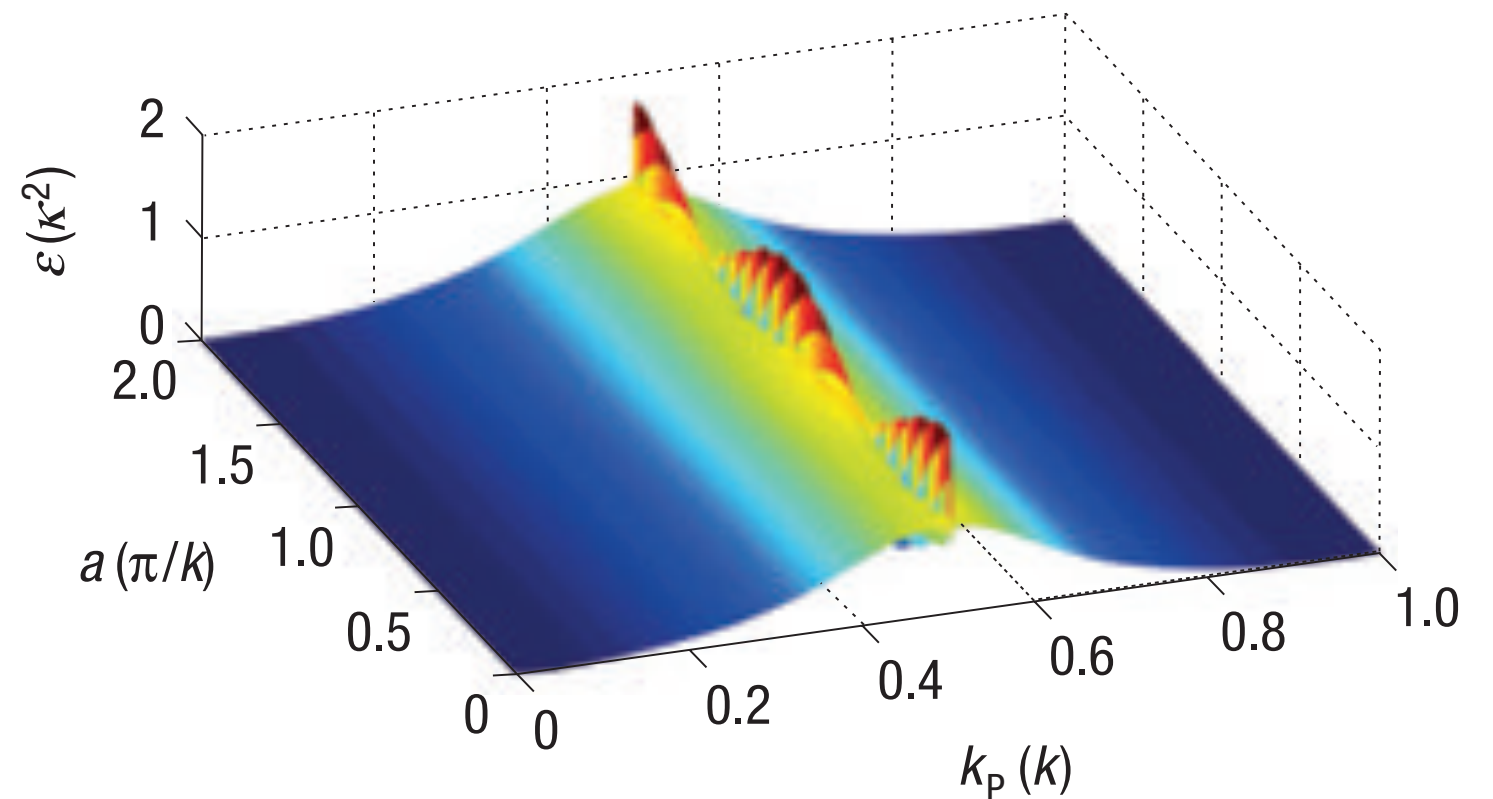
# Quantum Polarization Spectroscopy

Quantum non-demolition detection of strongly correlated systems

KAI ECKERT<sup>1</sup>, ORIOL ROMERO-ISART<sup>1</sup>, MIRTA RODRIGUEZ<sup>2</sup>, MACIEJ LEWENSTEIN<sup>2,3</sup>,  
EUGENE S. POLZIK<sup>4</sup> AND ANNA SANPERA<sup>1,3\*</sup>



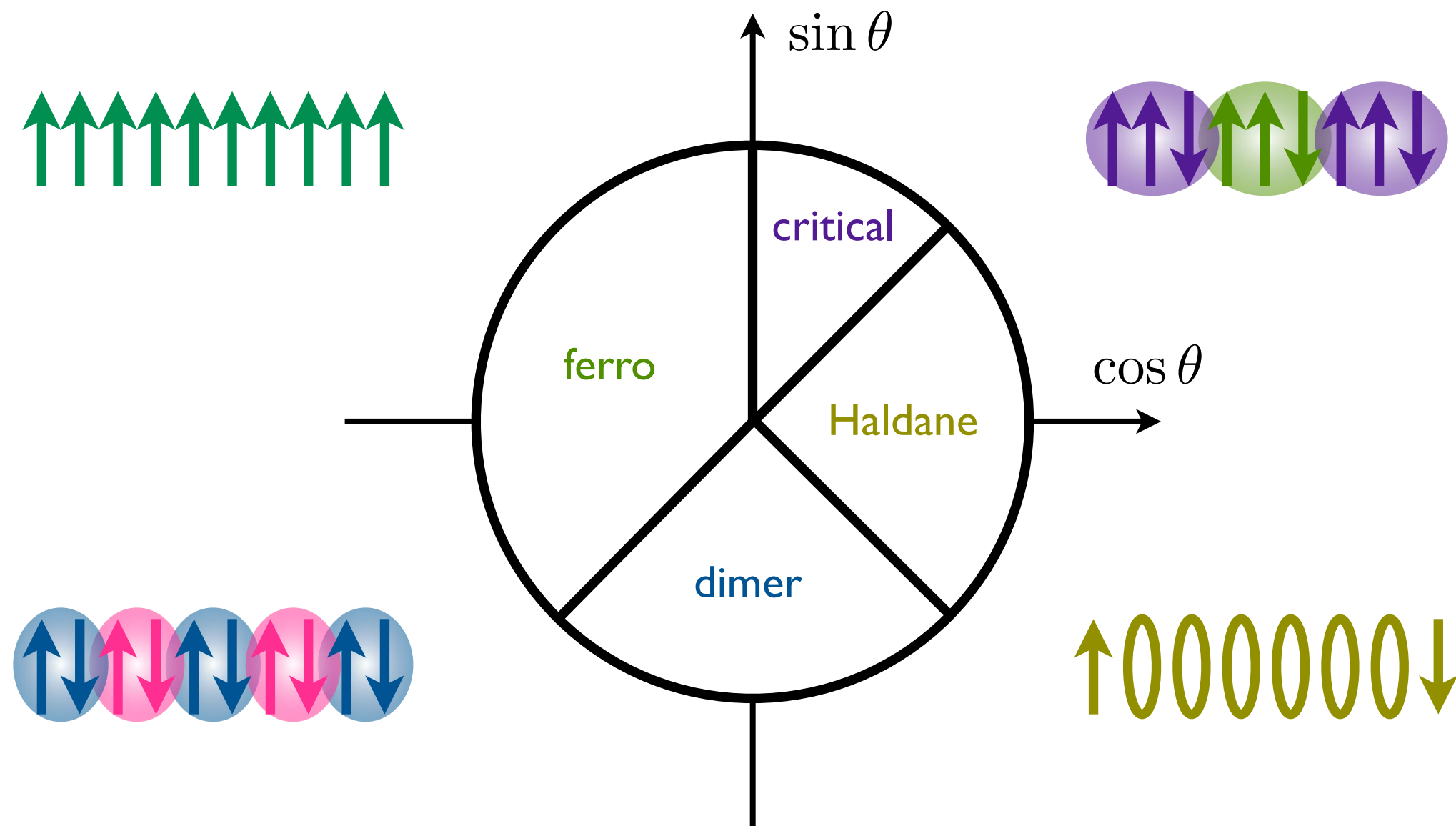
**d**



Eckert, Nat. Phys. 4, 50 (2008)

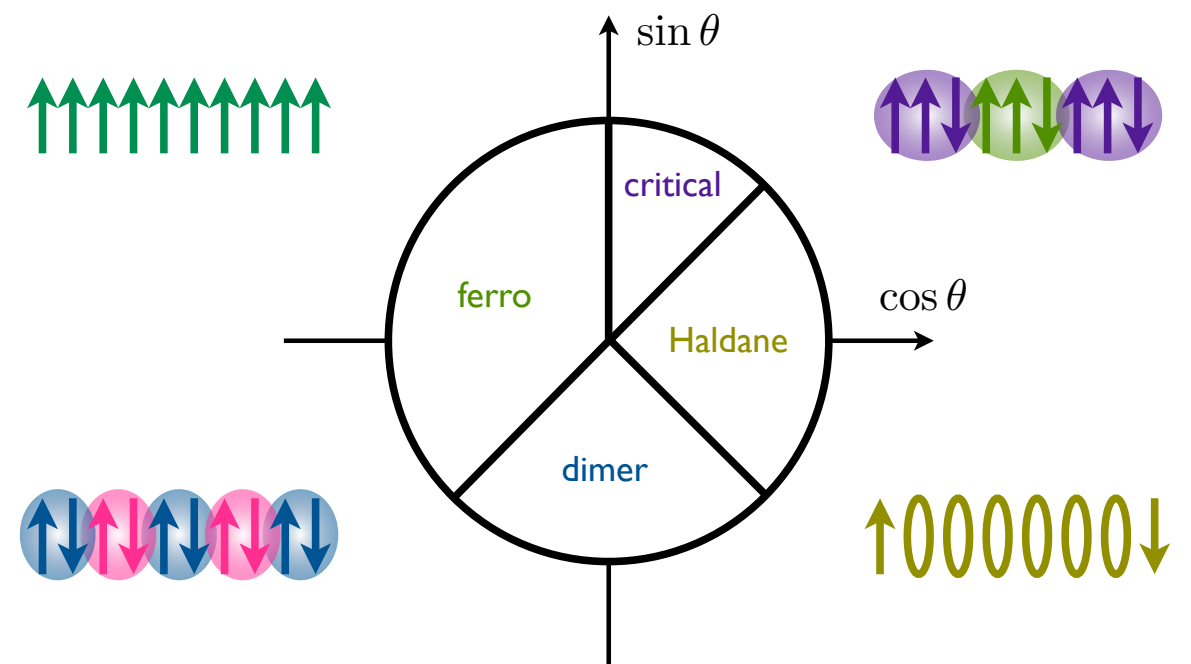
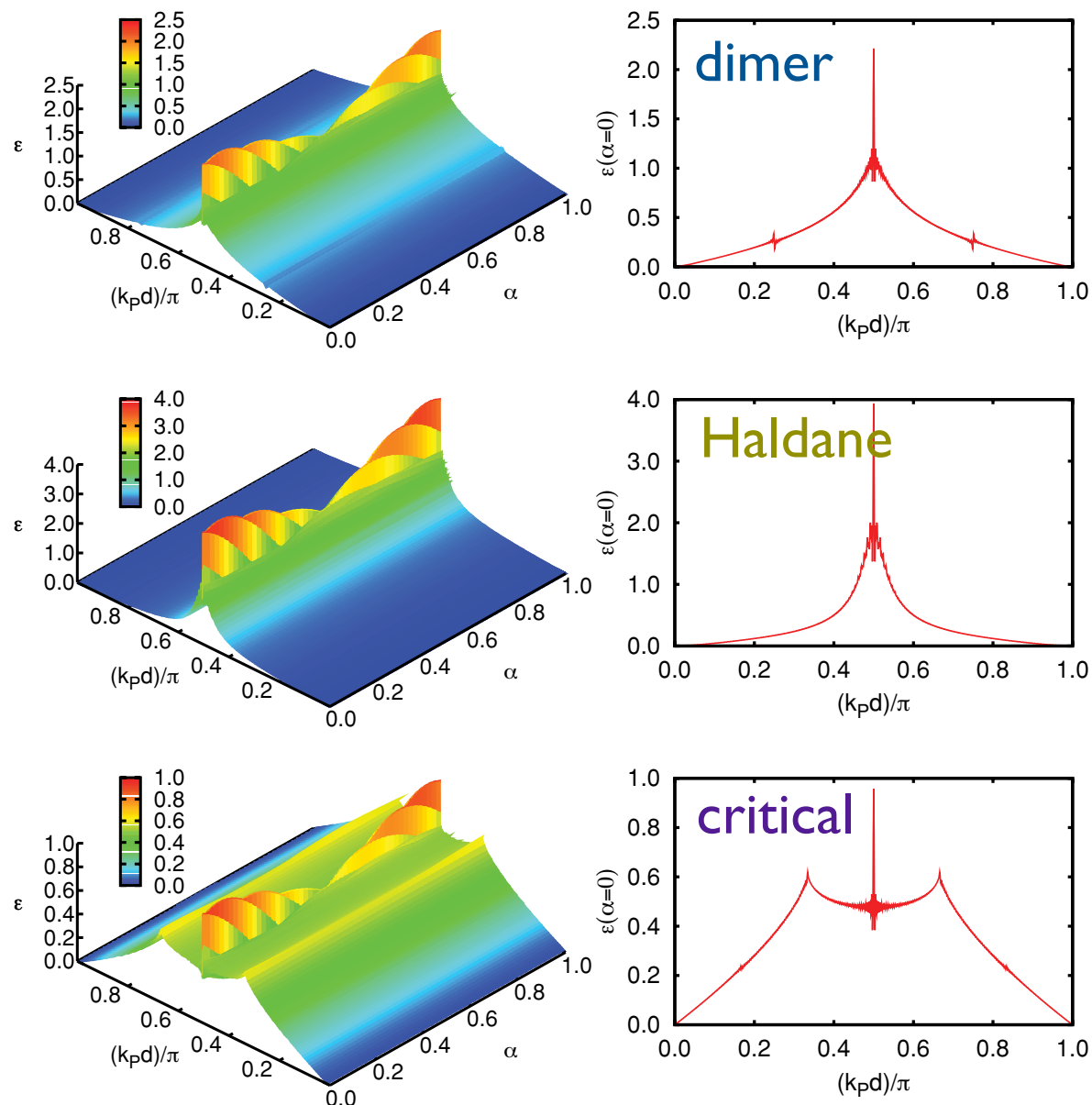
Example: 1D chain of spin-1 atoms described by the bilinear-biquadratic Hamiltonian

$$H = \sum_i \cos \theta \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sin \theta (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2$$



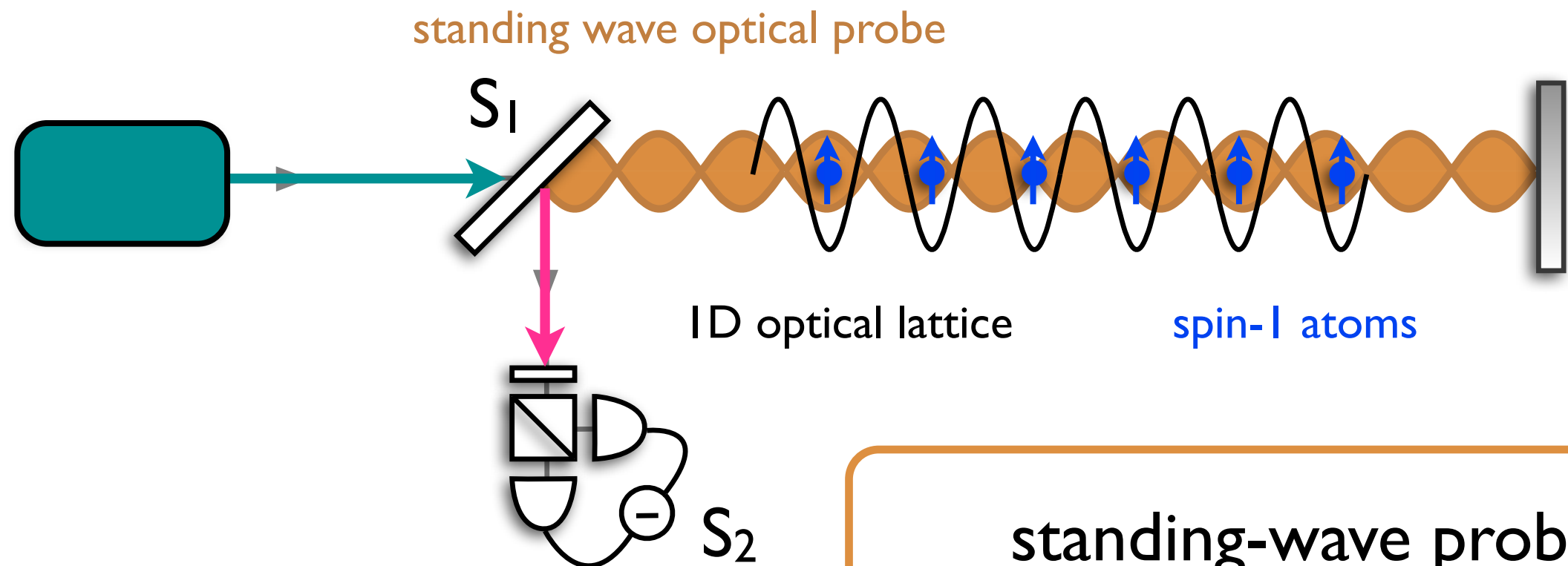
# Example: 1D chain of spin-1 atoms described by the bilinear-biquadratic Hamiltonian

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De Chiara, PRA 83, 021604 (2011)

# Quantum Polarization Spectroscopy



atom-light interaction

$$H_p = \Omega_p \sum_i c_i(k_p) J_{z,i} S_3$$

$$J_{\alpha,i} \equiv \sum_{n=1}^{n_a} j_{\alpha,i}^{(n)} \quad \Omega_p \equiv \frac{\sigma(\Delta)}{A}$$

standing-wave probe

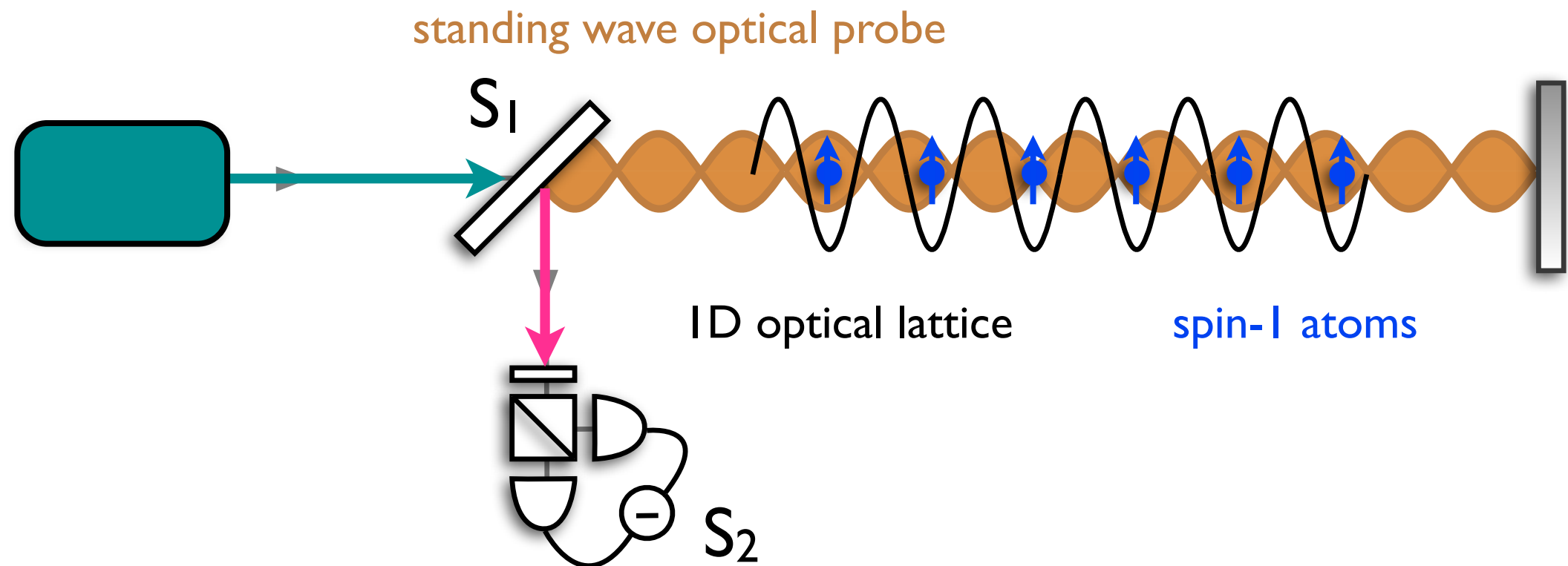
$$c_i(k_p) = (1 + \cos(2k_p r_i))/2$$

measurement

$$S_2^{(\text{out})} = S_2^{(\text{in})} + \kappa_p \sum_i c_i(k_p) J_{z,i}$$



# Quantum Polarization Spectroscopy



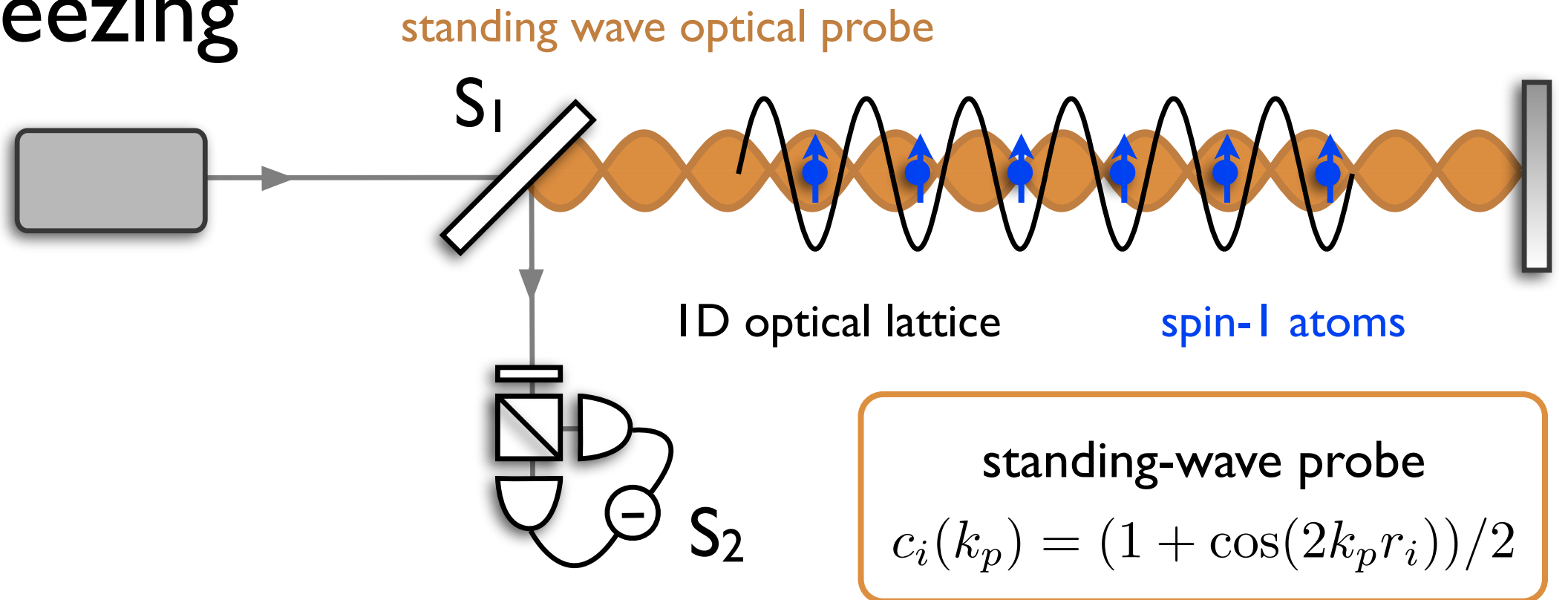
detected variance

$$\Delta^2 S_2^{(\text{out})} = \Delta^2 S_2^{(\text{in})} + \kappa_p^2 \sum_{i,j} c_i(k_p) c_j(k_p) G_{ij}$$

spin correlation function

$$G_{ij} \equiv \langle J_z(r_i) J_z(r_j) \rangle - \langle J_z(r_i) \rangle \langle J_z(r_j) \rangle$$

# Spin Squeezing



squeeze atomic spin wave

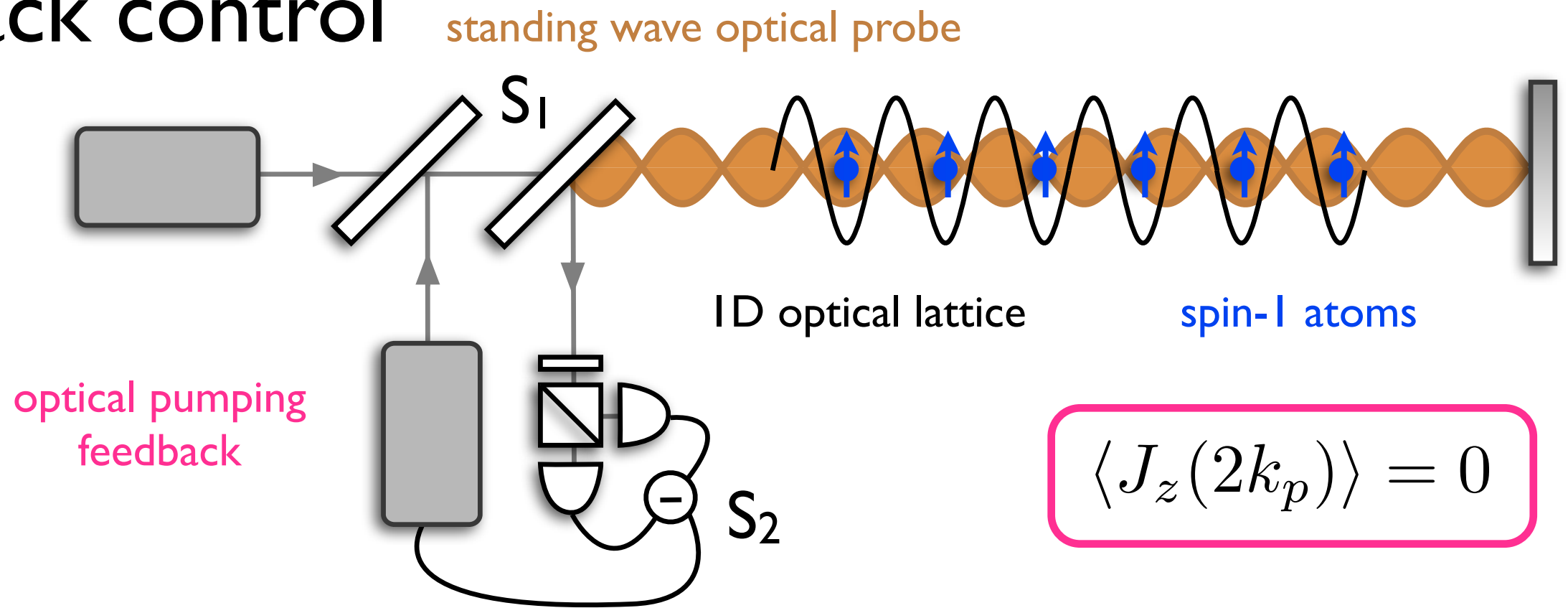
$$\Delta^2 J_z^{(\text{out})}(\pm 2k_p) \simeq \frac{\Delta^2 J_z^{(\text{in})}(\pm 2k_p)}{1 + \kappa_p^2}$$

Hauke, PRA 87, 021601 (2013)

$$J_\alpha(k) \equiv \frac{1}{\sqrt{n_s}} \sum_i J_{\alpha,i} \exp(ikr_i)$$



# Feedback control



$$\langle J_z(2k_p) \rangle = 0$$

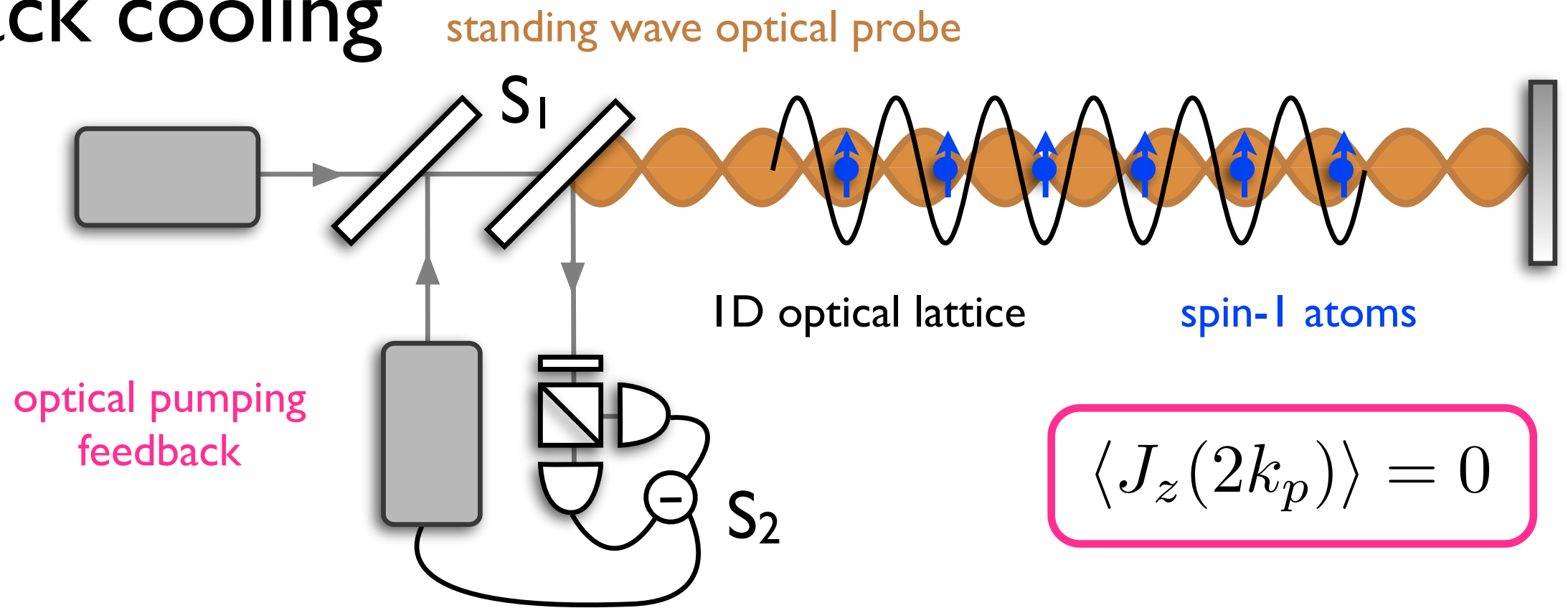
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Hauke, PRA 87, 021601 (2013)

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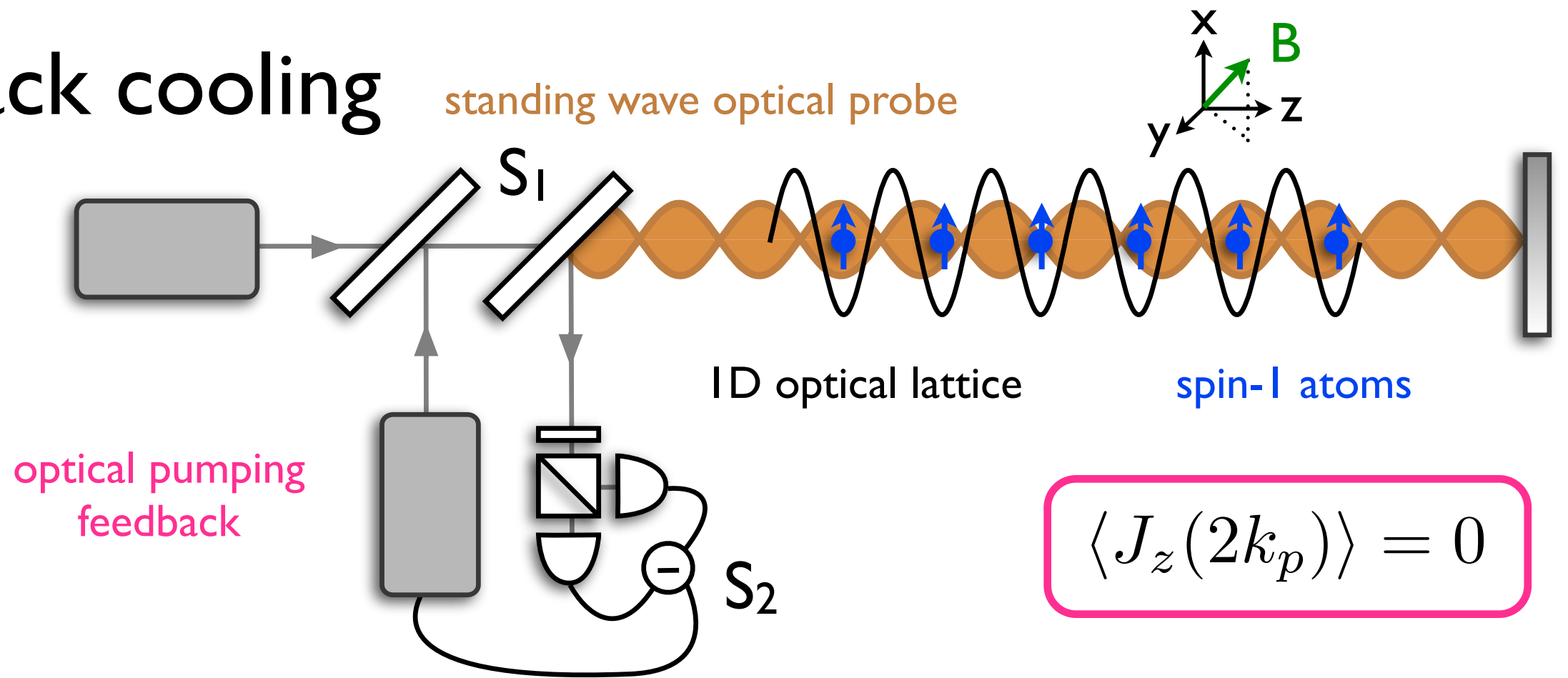
# Feedback cooling



Heisenberg uncertainty relation

$$\Delta J_\alpha(k_1) \Delta J_\beta(k_2) \geq \frac{1}{4} |\langle J_\gamma(k_1 + k_2) \rangle|$$

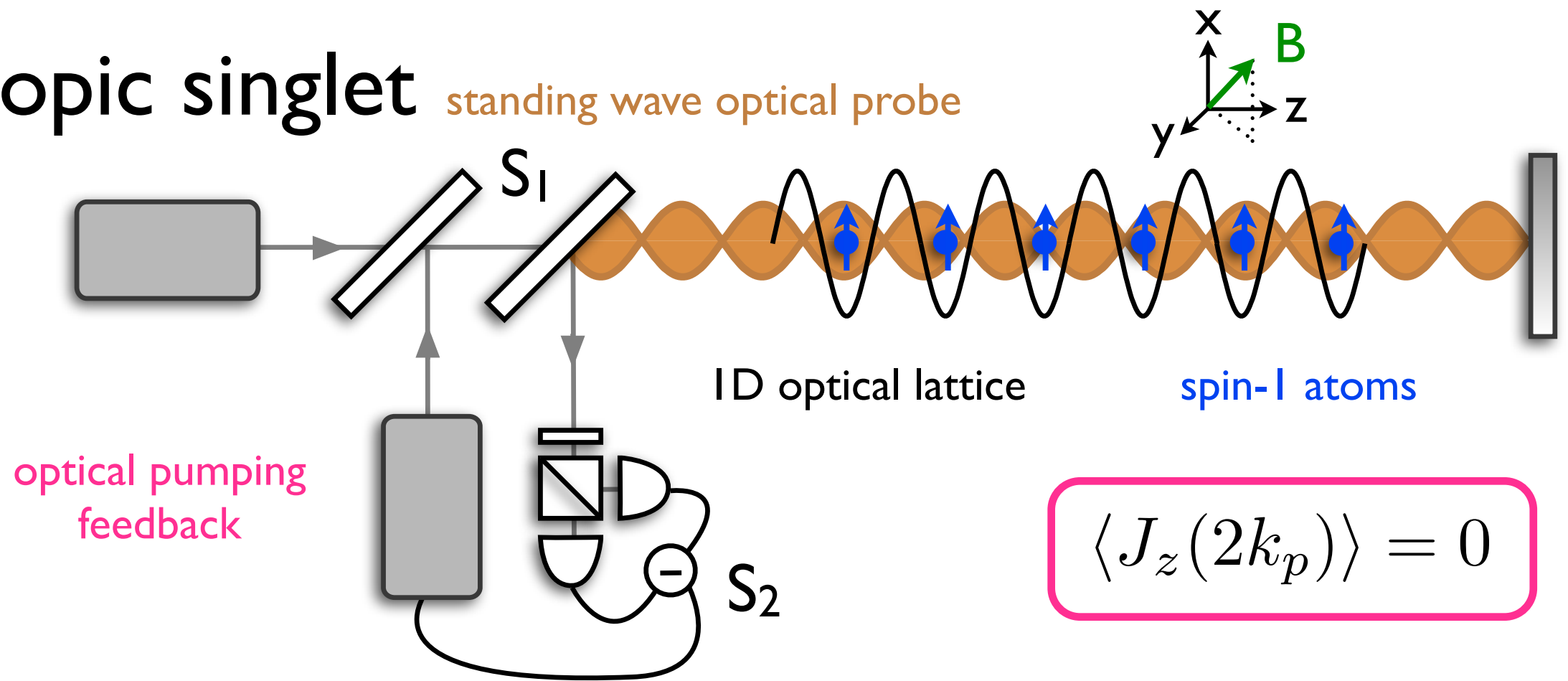
# Feedback cooling



Heisenberg uncertainty relation

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# Macroscopic singlet standing wave optical probe



macroscopic spin singlet

$$\Delta^2 J_\alpha(2k_p) \rightarrow 0 \quad \& \quad \langle J_\alpha(2k_p) \rangle = 0$$

$$|\mathbf{J}(2k_p)| \rightarrow 0$$

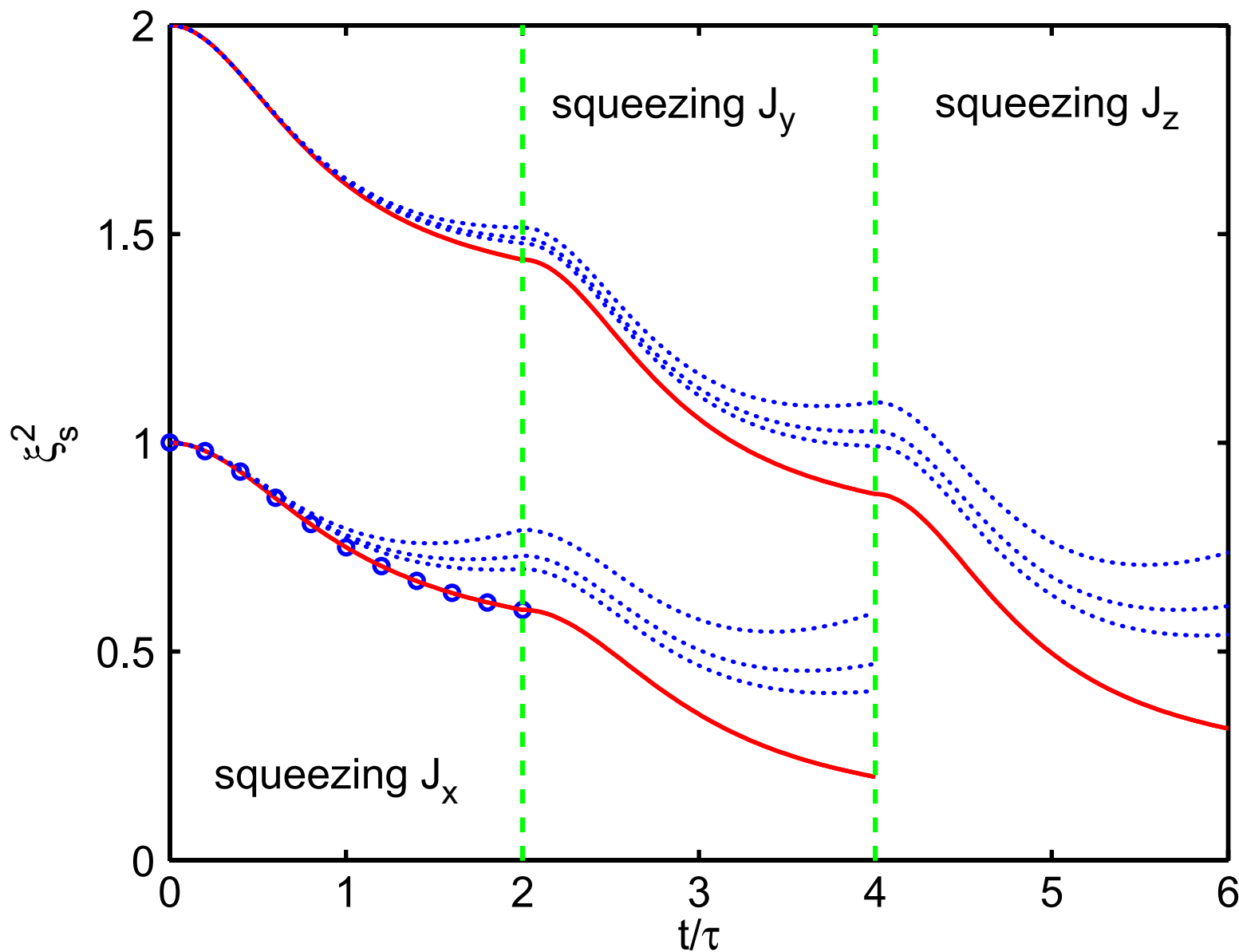
Hauke, PRA 87, 021601 (2013)

G.Tóth, NJP 12, 053007 (2010)

I. Urizar-Lanz, PRA 88, 013626 (2013)

# Generation of macroscopic singlet states in atomic ensembles

Géza Tóth<sup>1,2,3,5</sup> and Morgan W Mitchell<sup>4</sup>



spin squeezing via  
QND measurement

$$\Delta^2 J_z^{(\text{out})} = \frac{\Delta^2 J_z^{(\text{in})}}{1 + \kappa}$$

+

quantum control

$$\langle J_z(2k_p) \rangle = 0$$

=

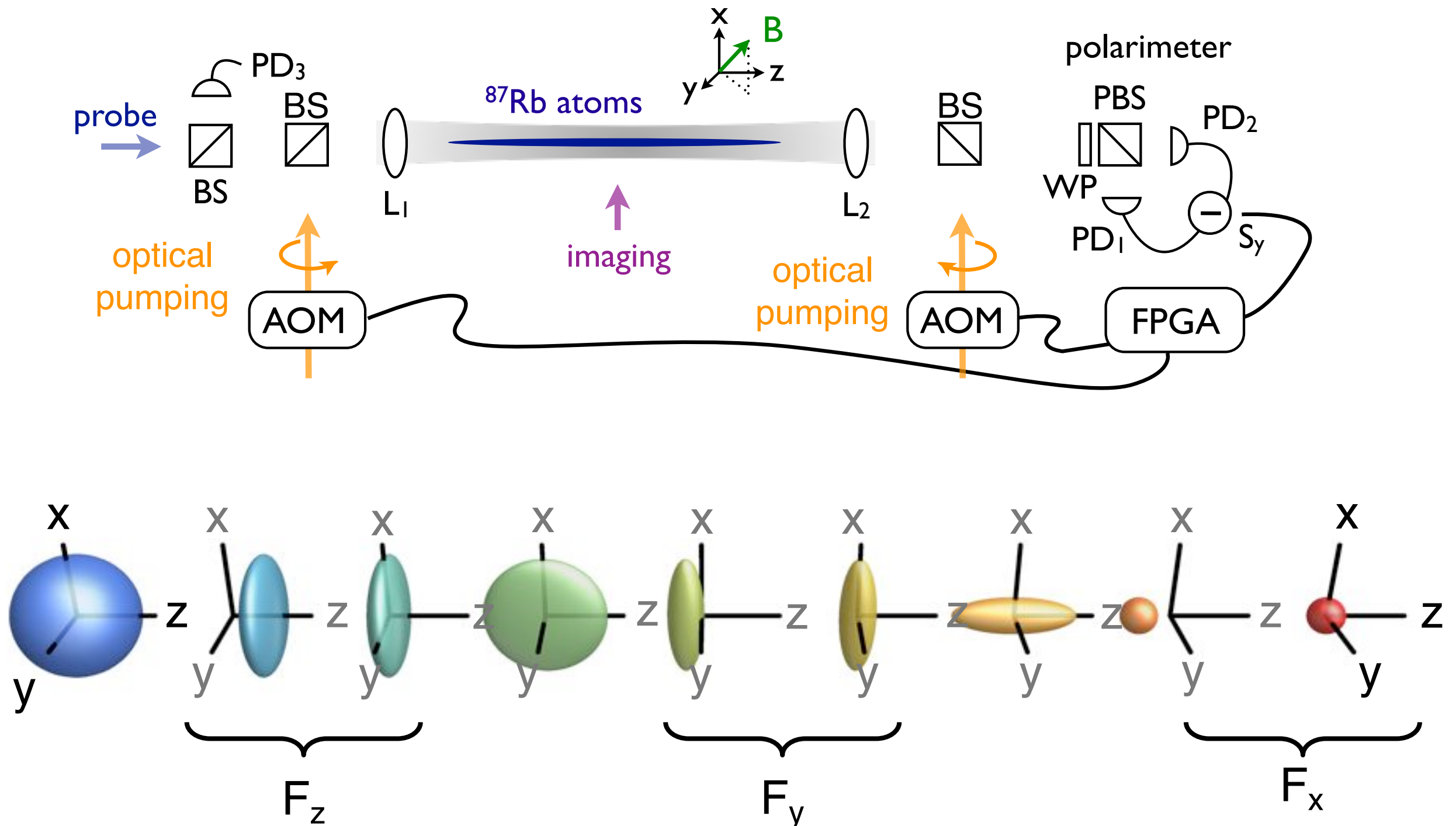
macroscopic singlet

$$|\mathbf{J}| \rightarrow 0$$

G.Tóth, NJP 12, 053007 (2010)

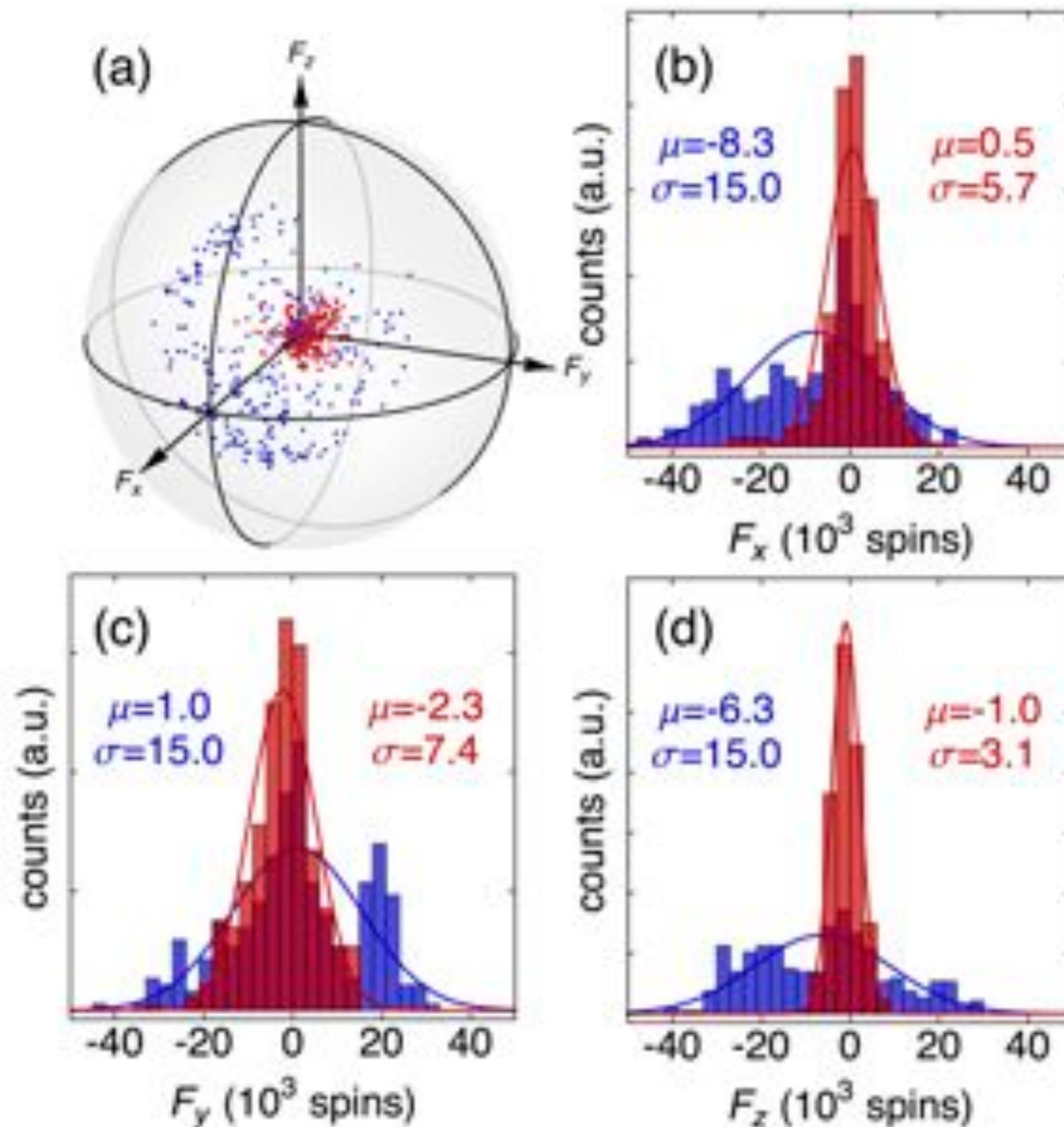
I. Urizar-Lanz, PRA 88, 013626 (2013)

# Feedback cooling of atomic spins

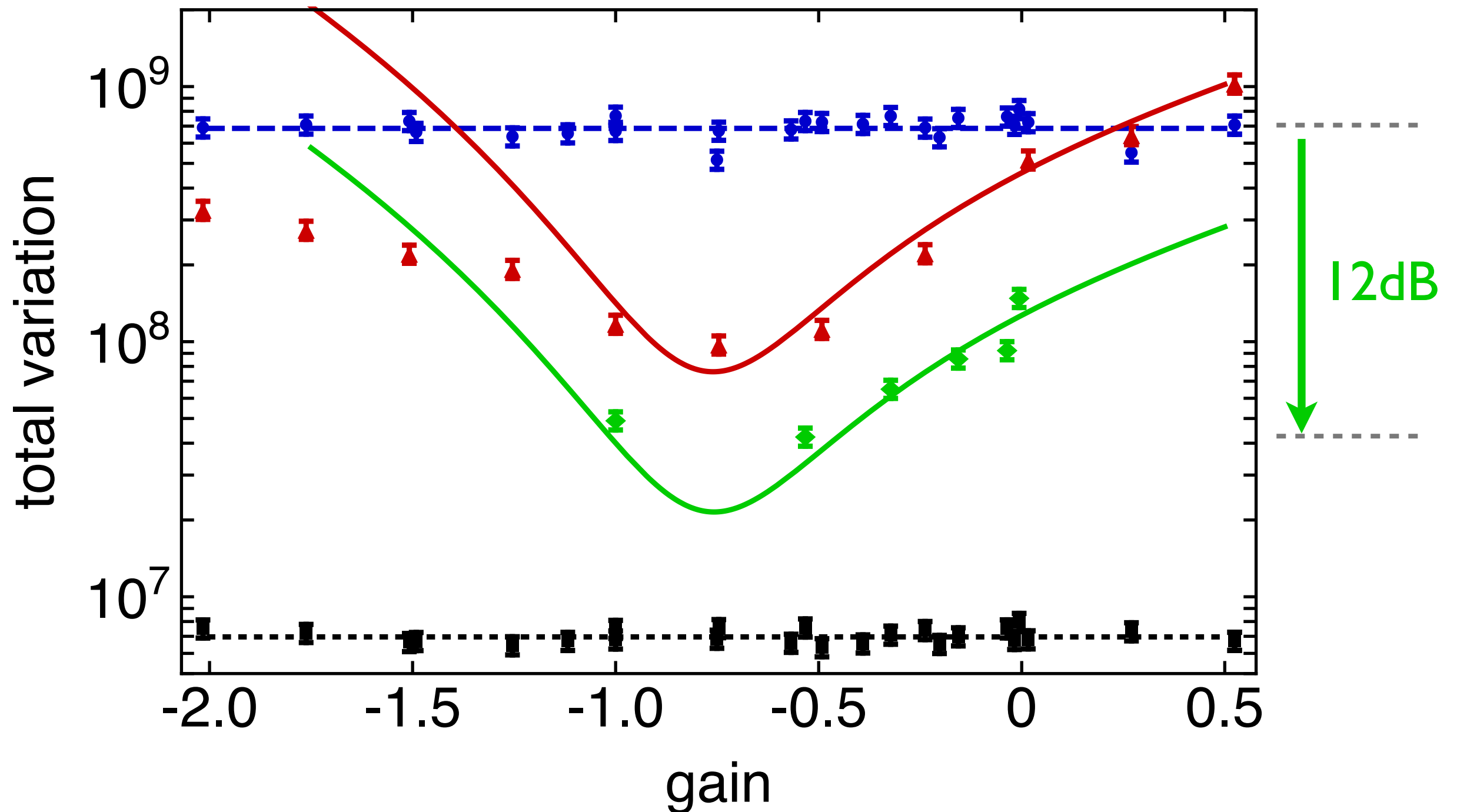




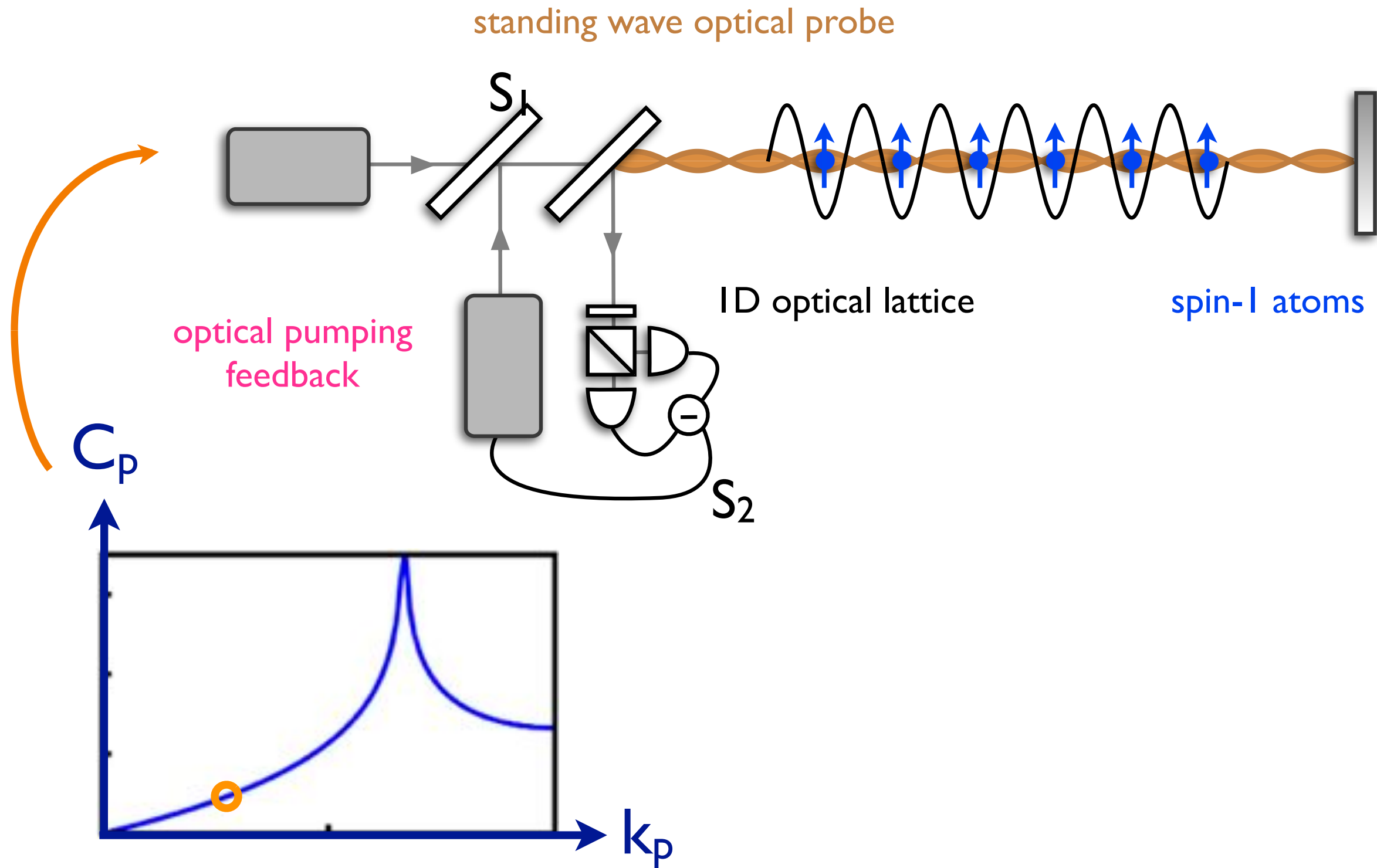
# Feedback cooling of atomic spins



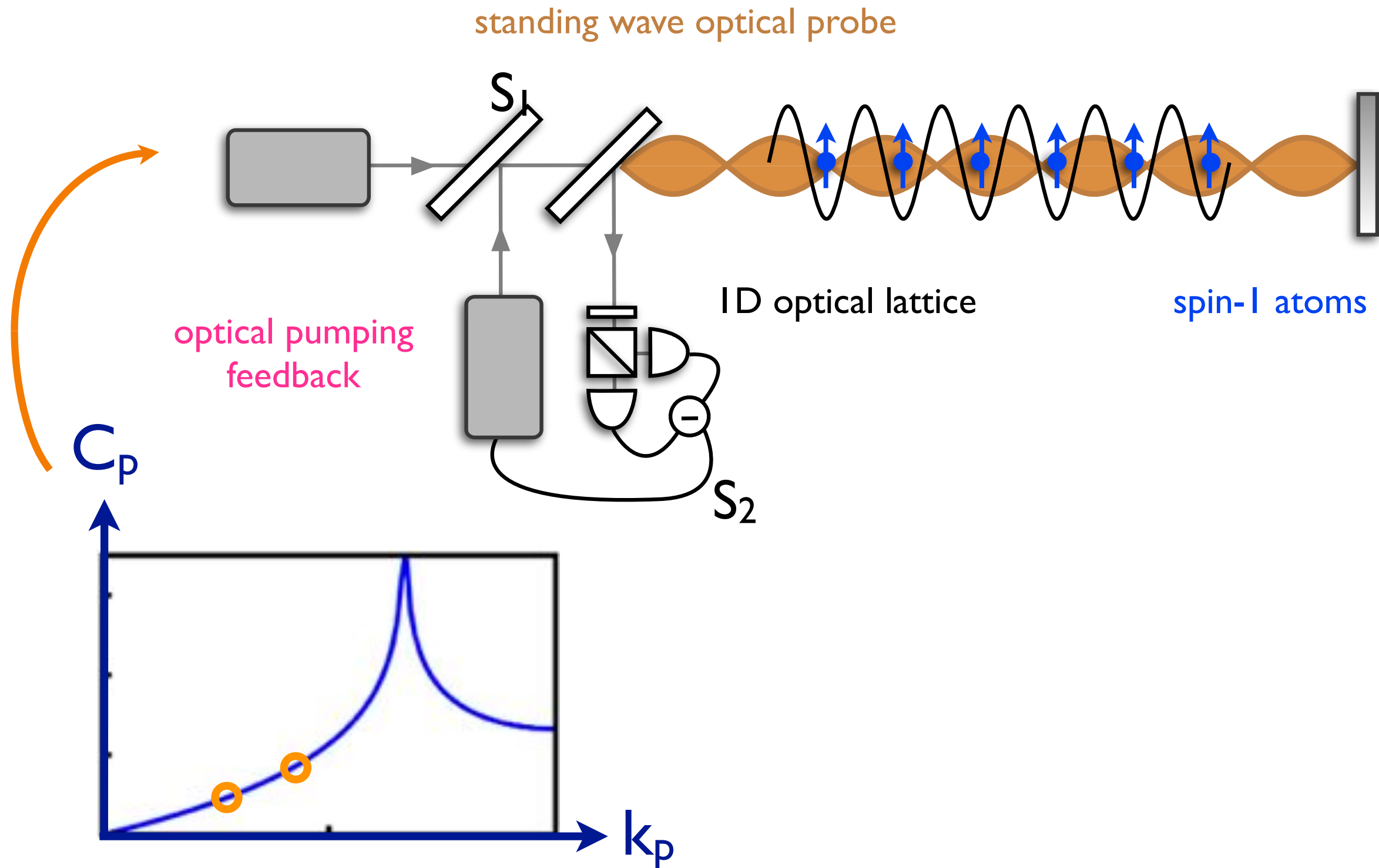
# Feedback cooling of atomic spins



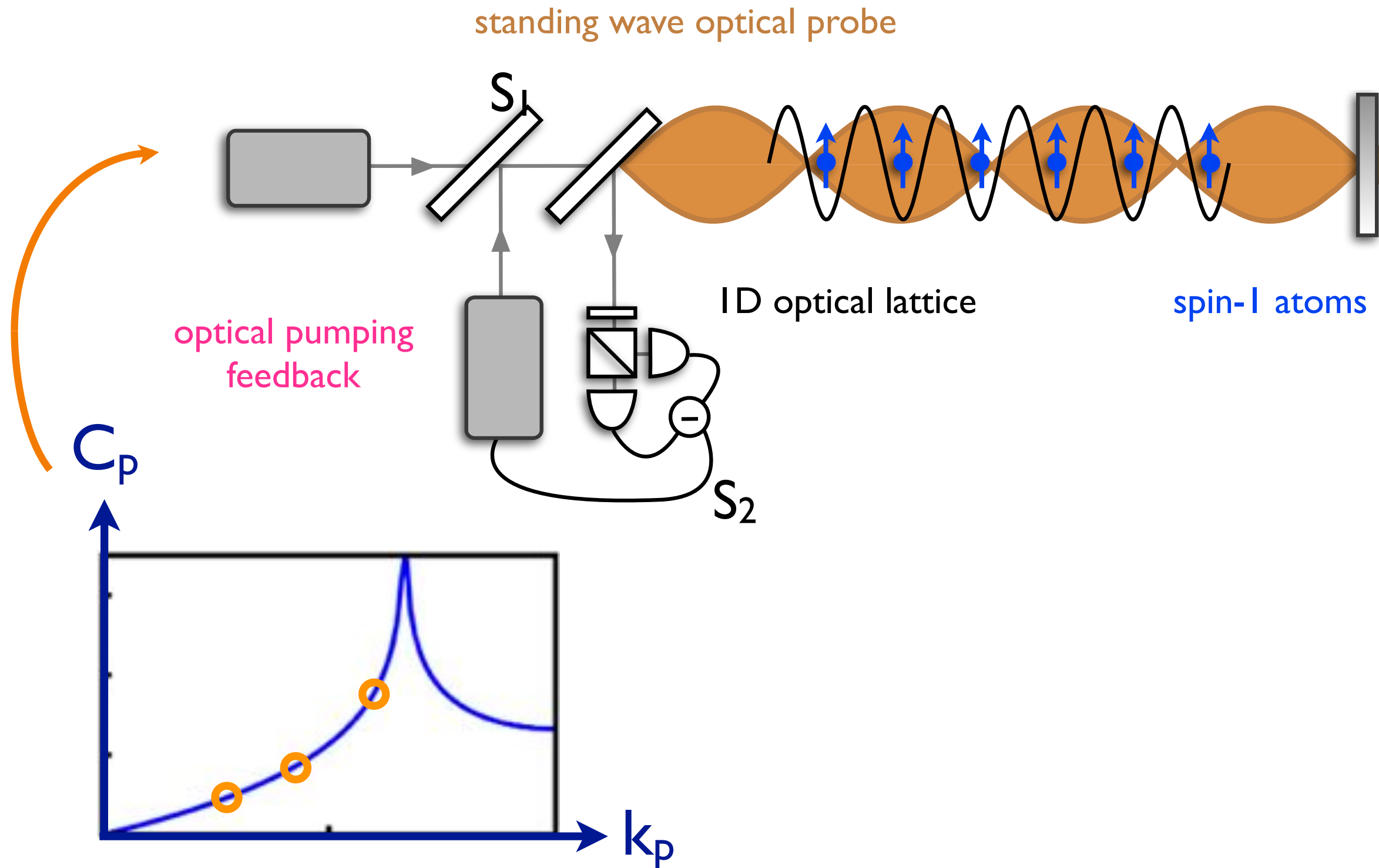
# Engineering quantum spin correlations



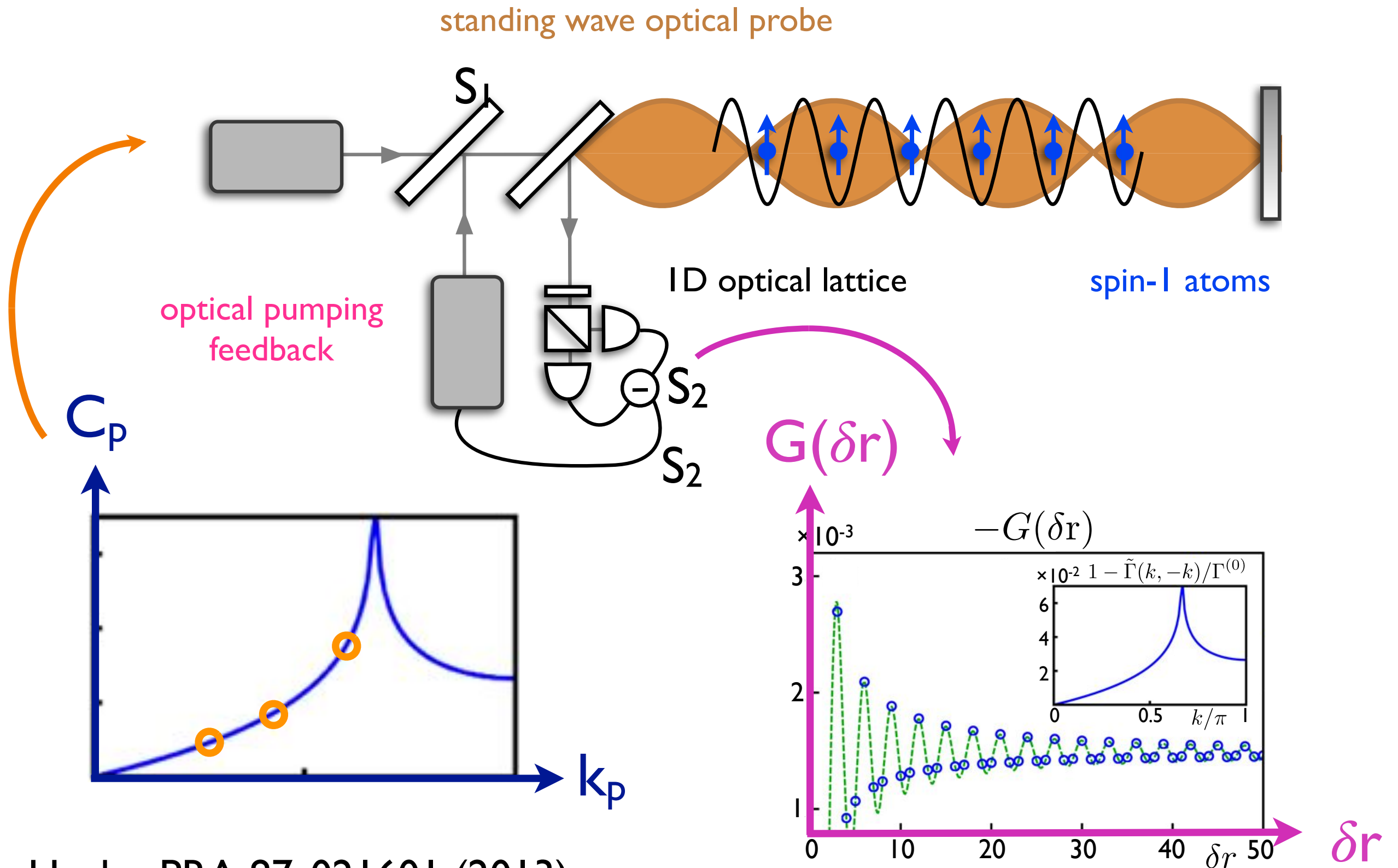
# Engineering quantum spin correlations



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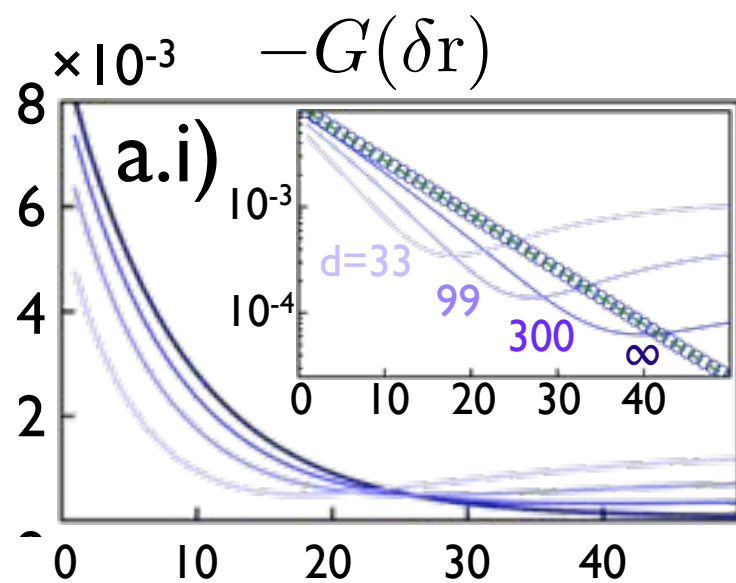
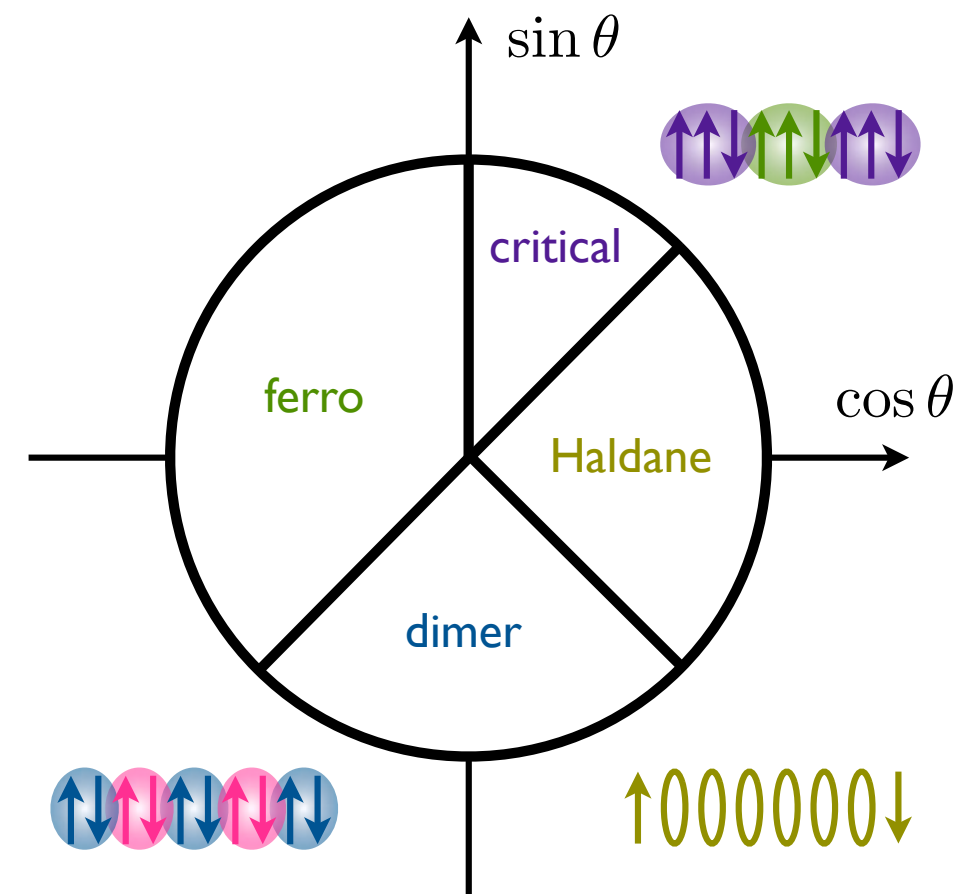
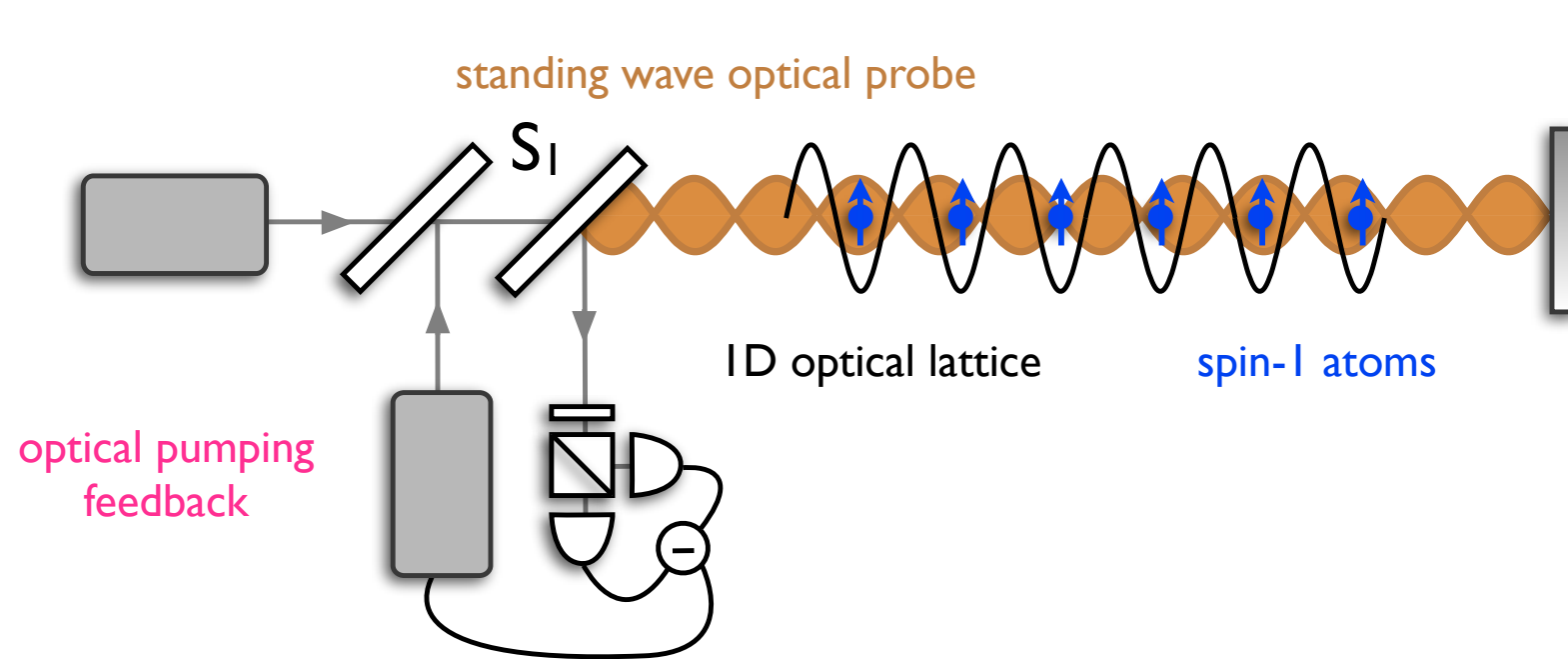


# Engineering quantum spin correlations



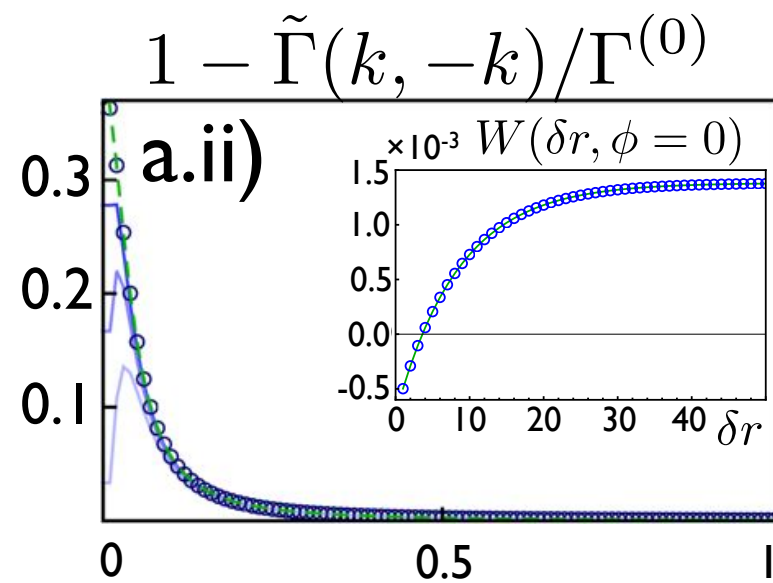
Hauke, PRA 87, 021601 (2013)



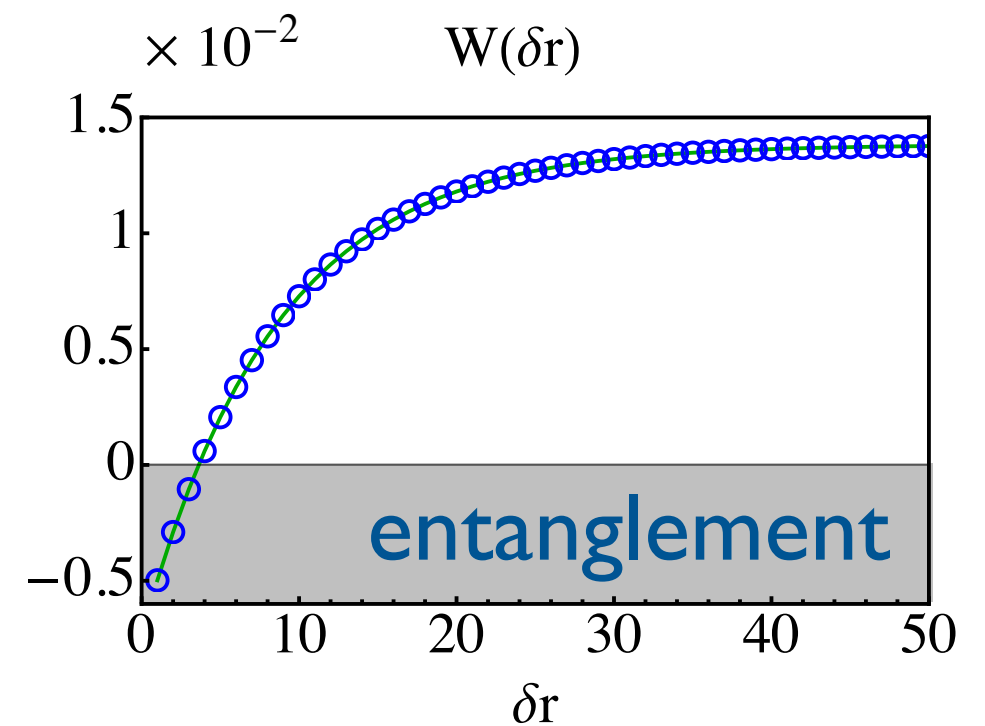


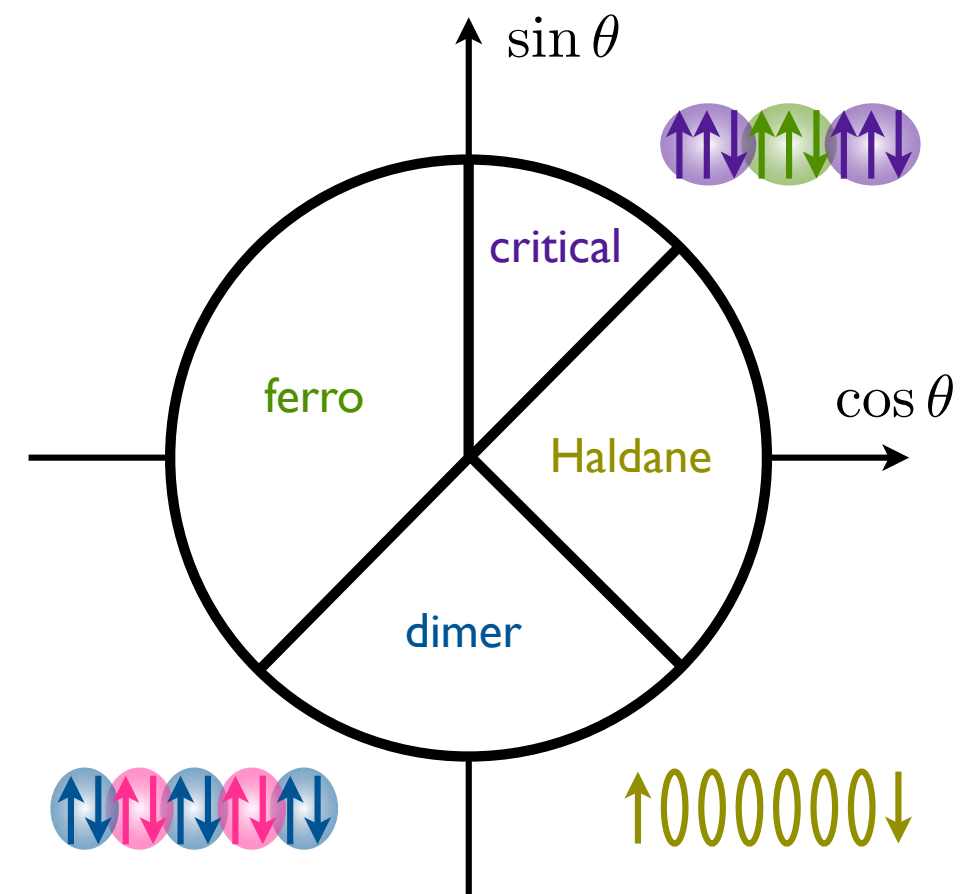
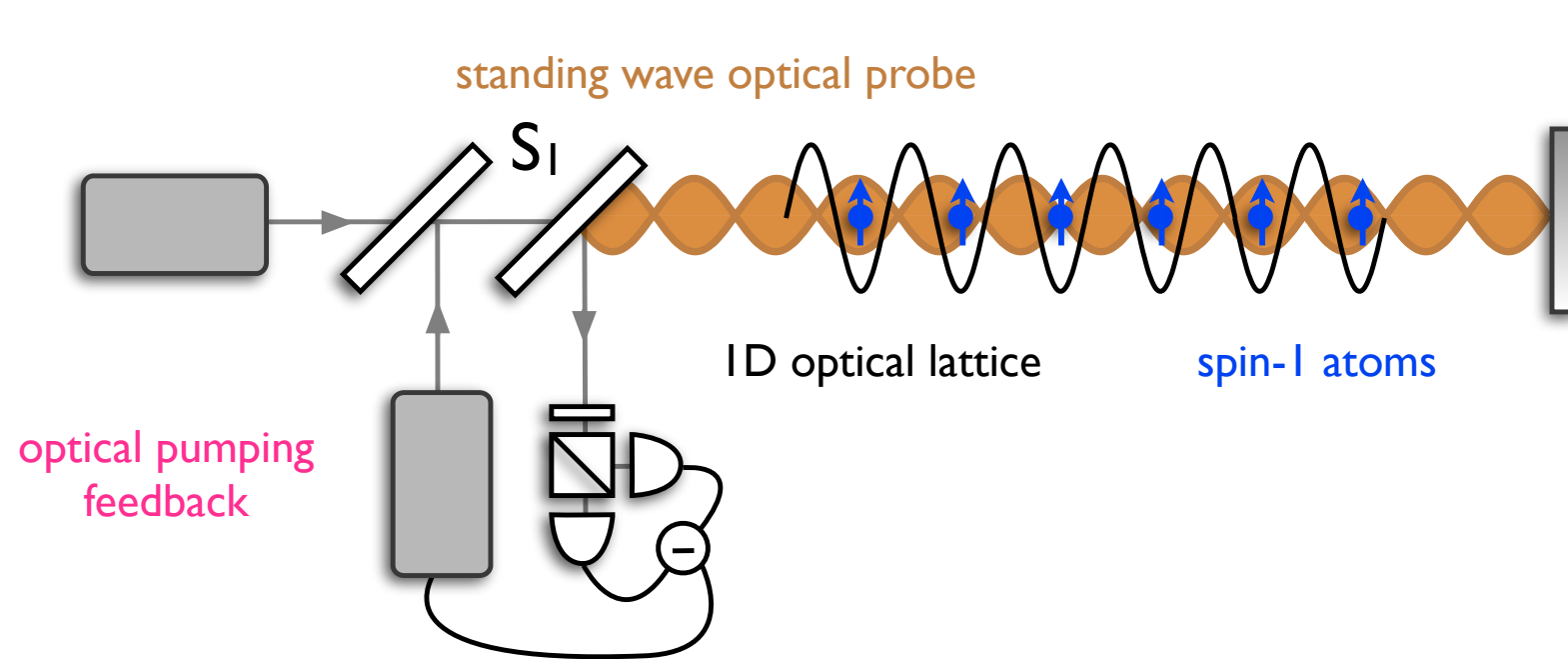
$\delta r$

dimer phase

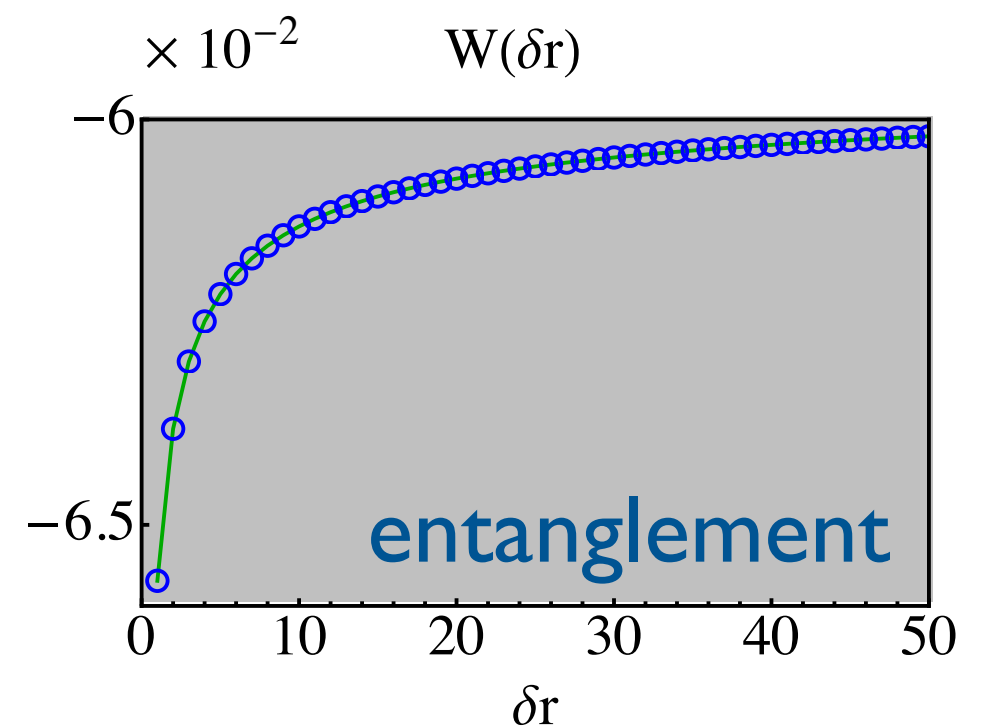
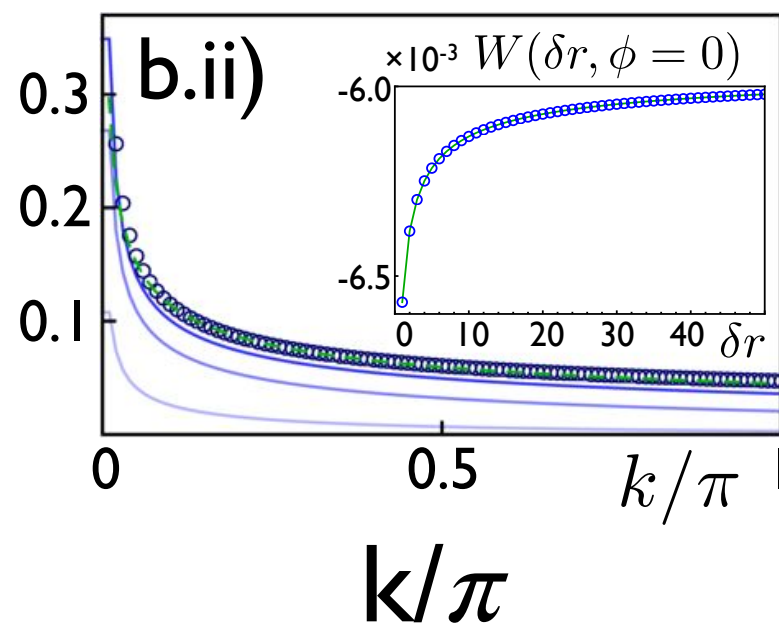
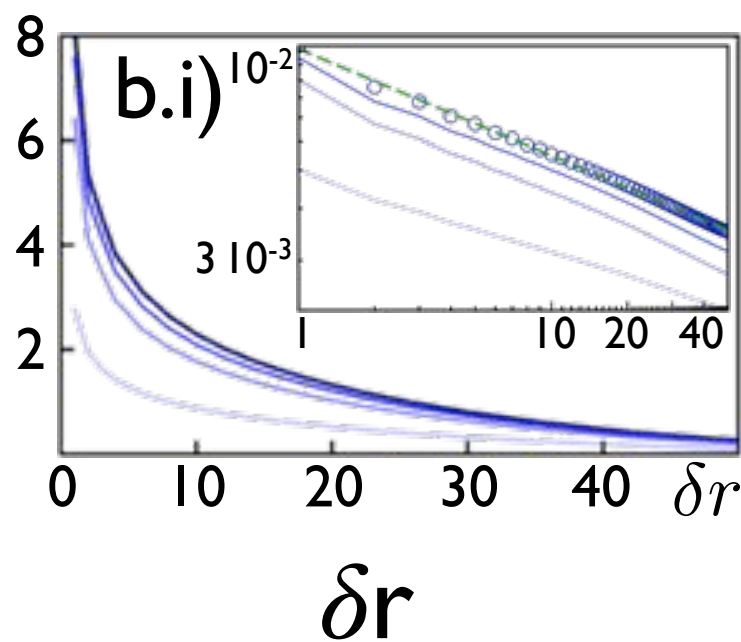


$k/\pi$

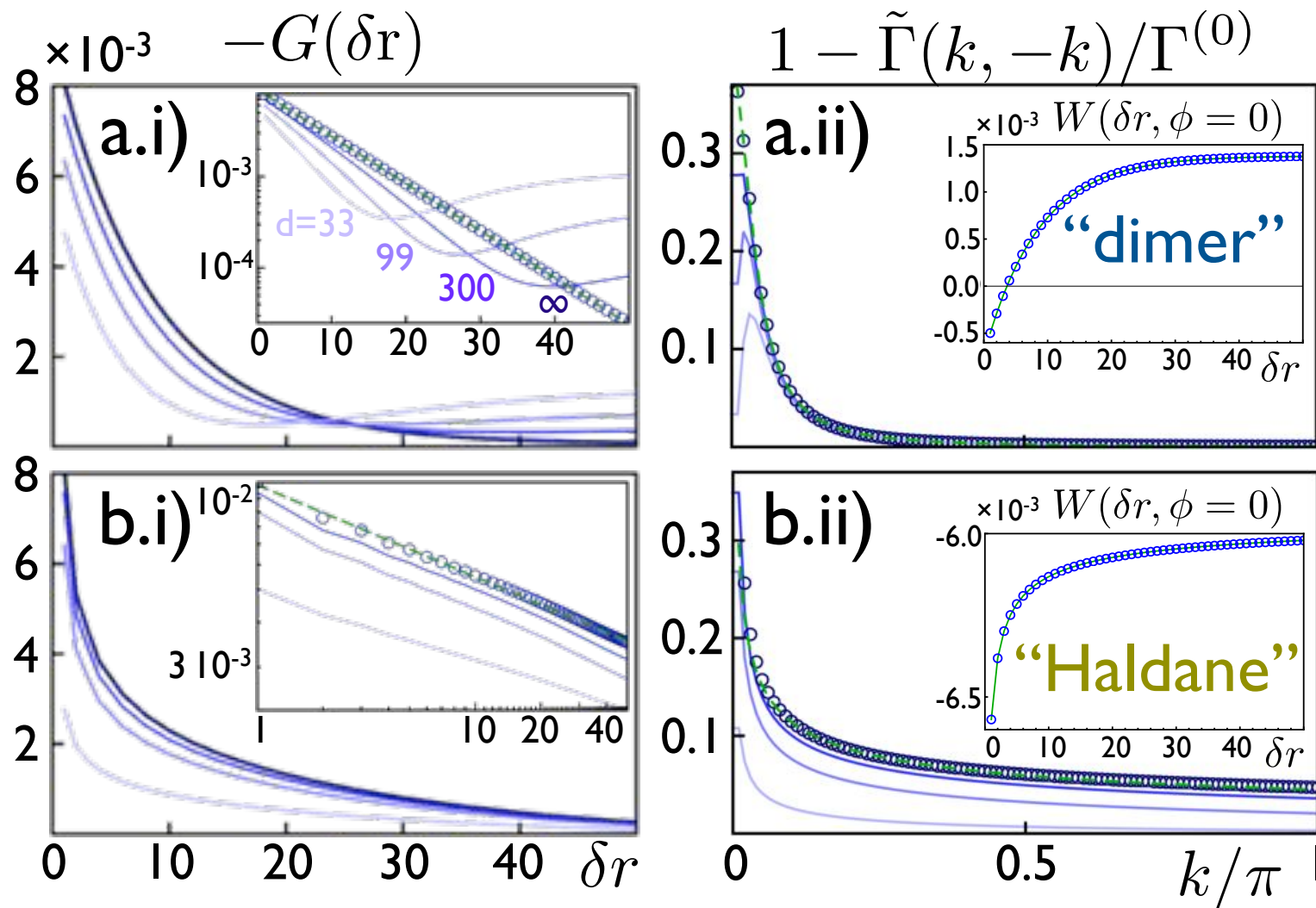
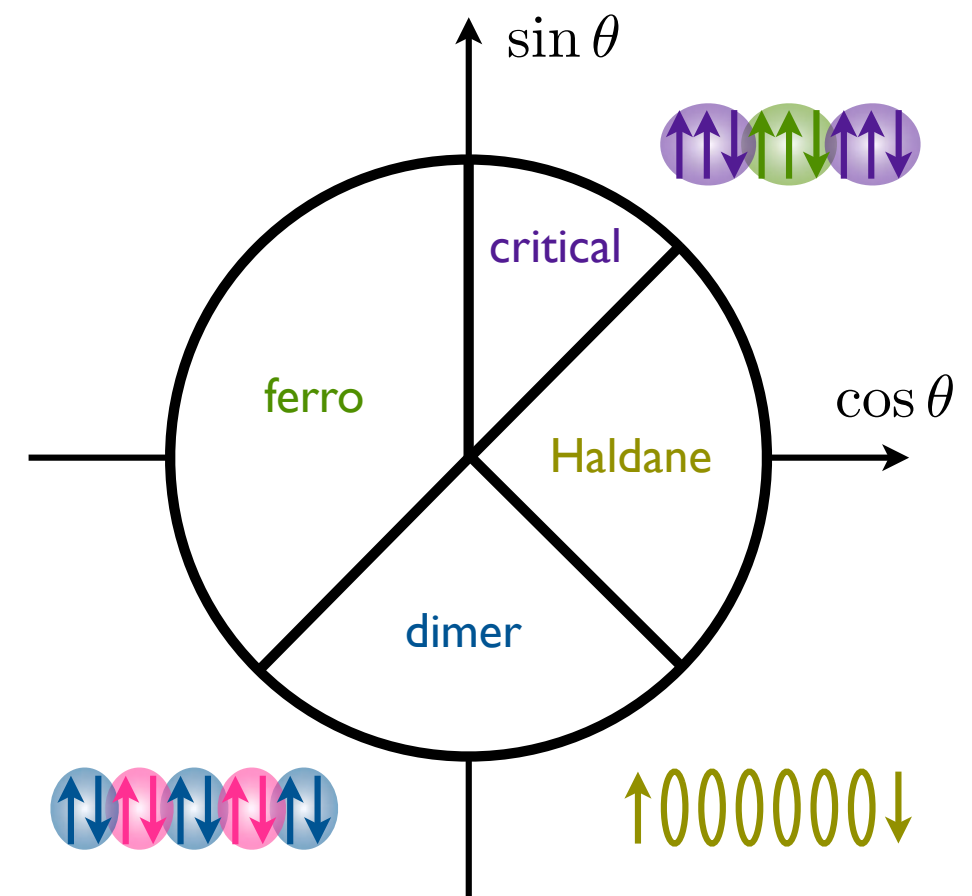
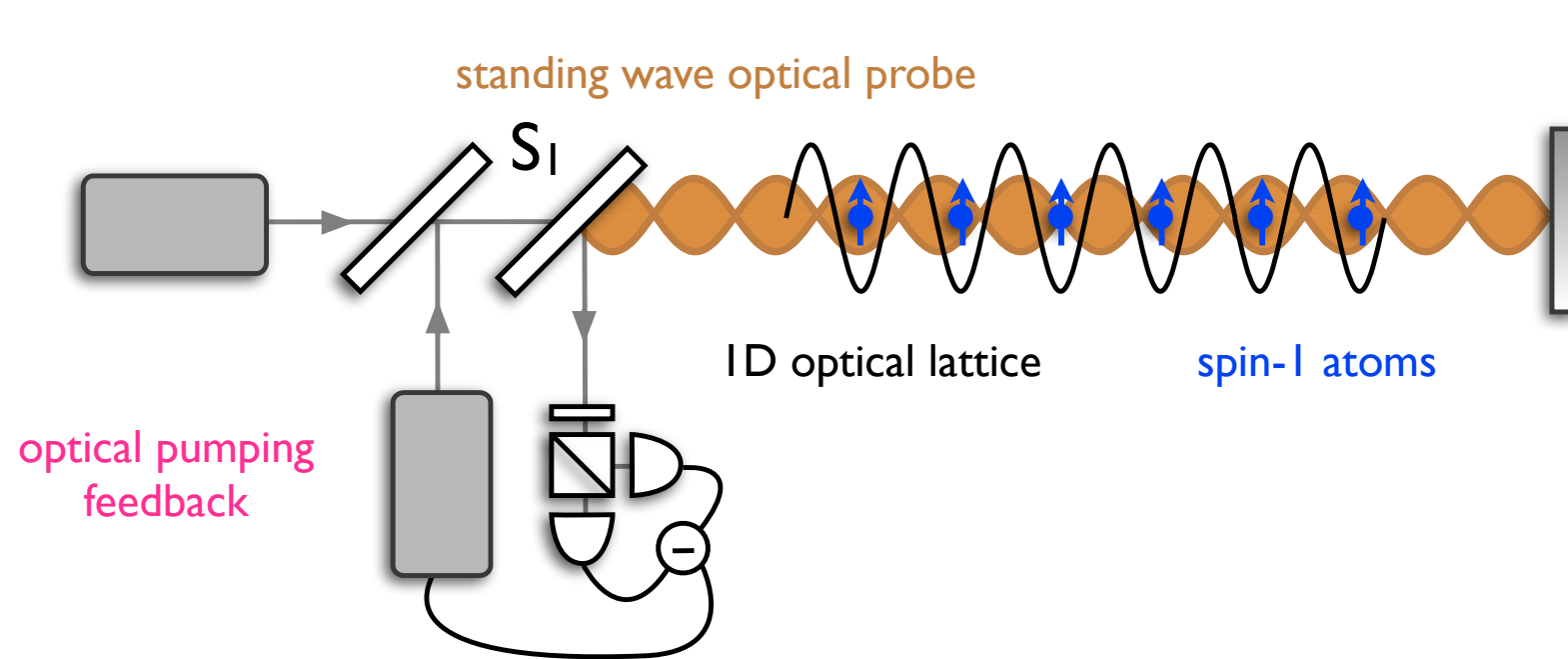


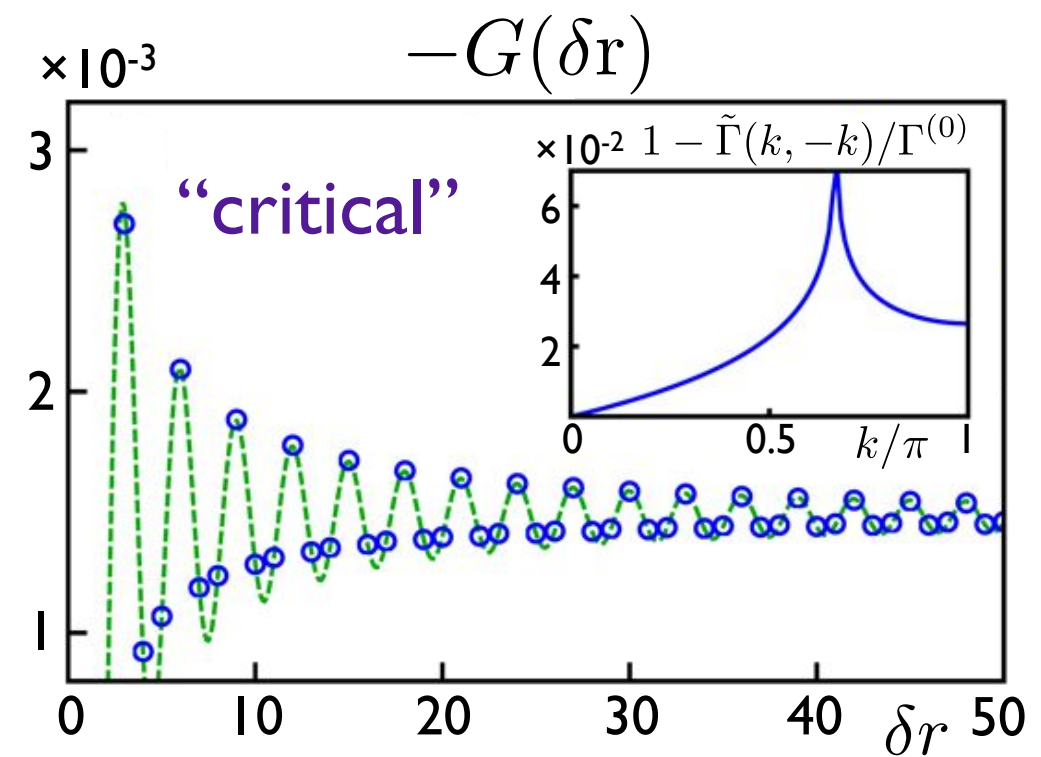
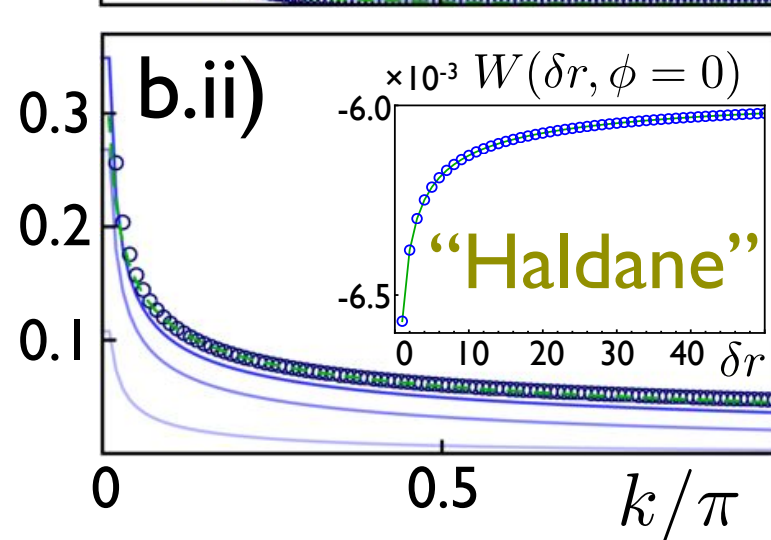
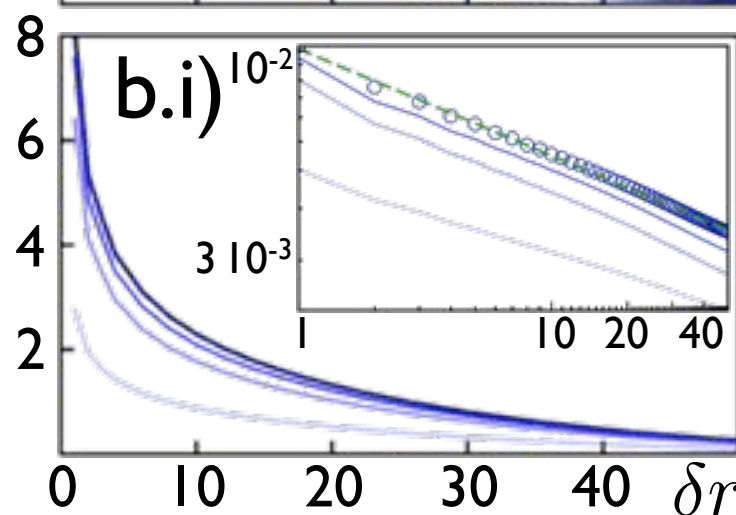
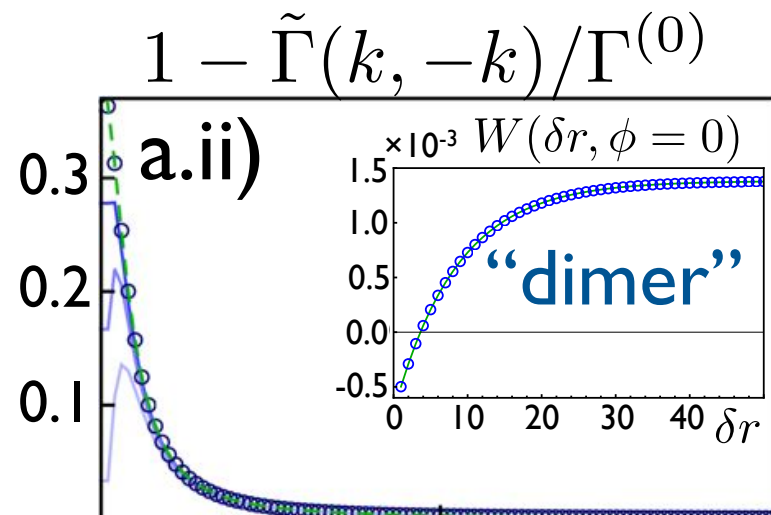
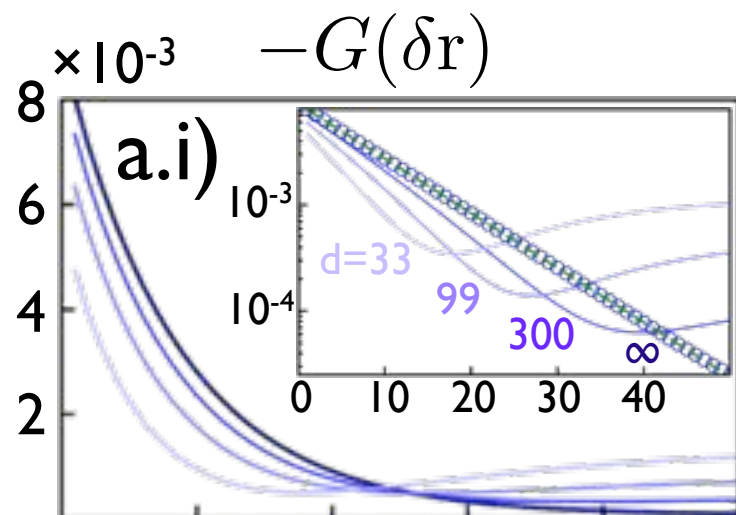
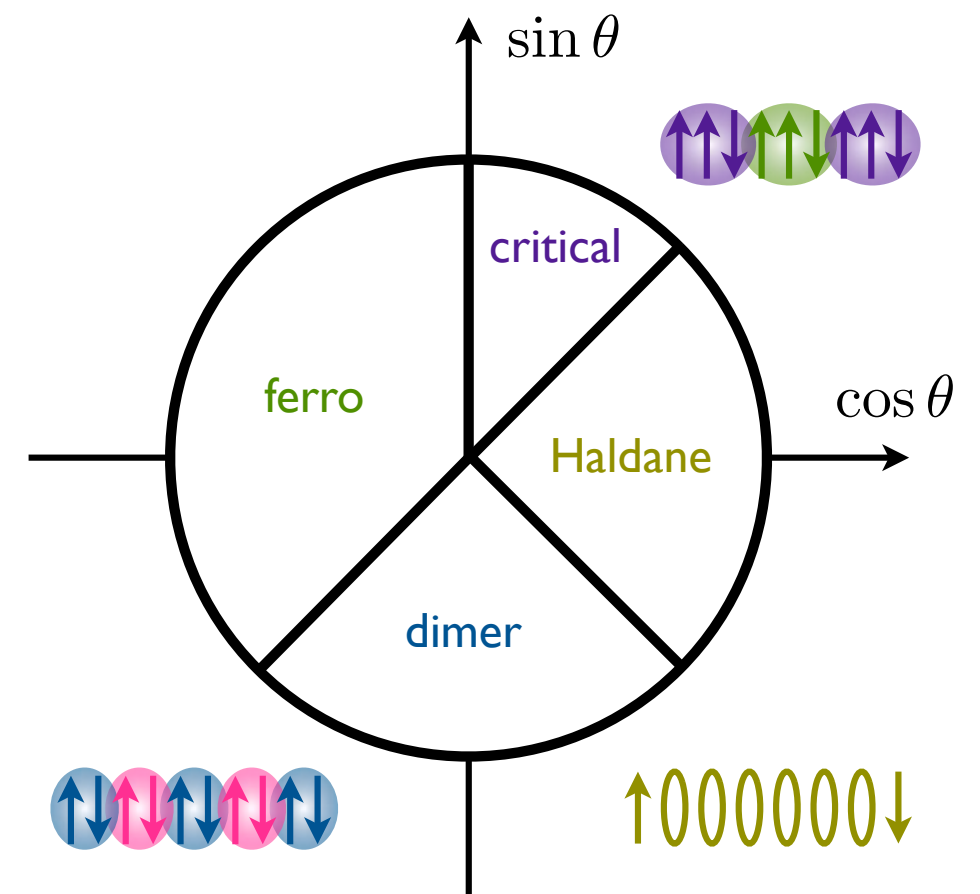
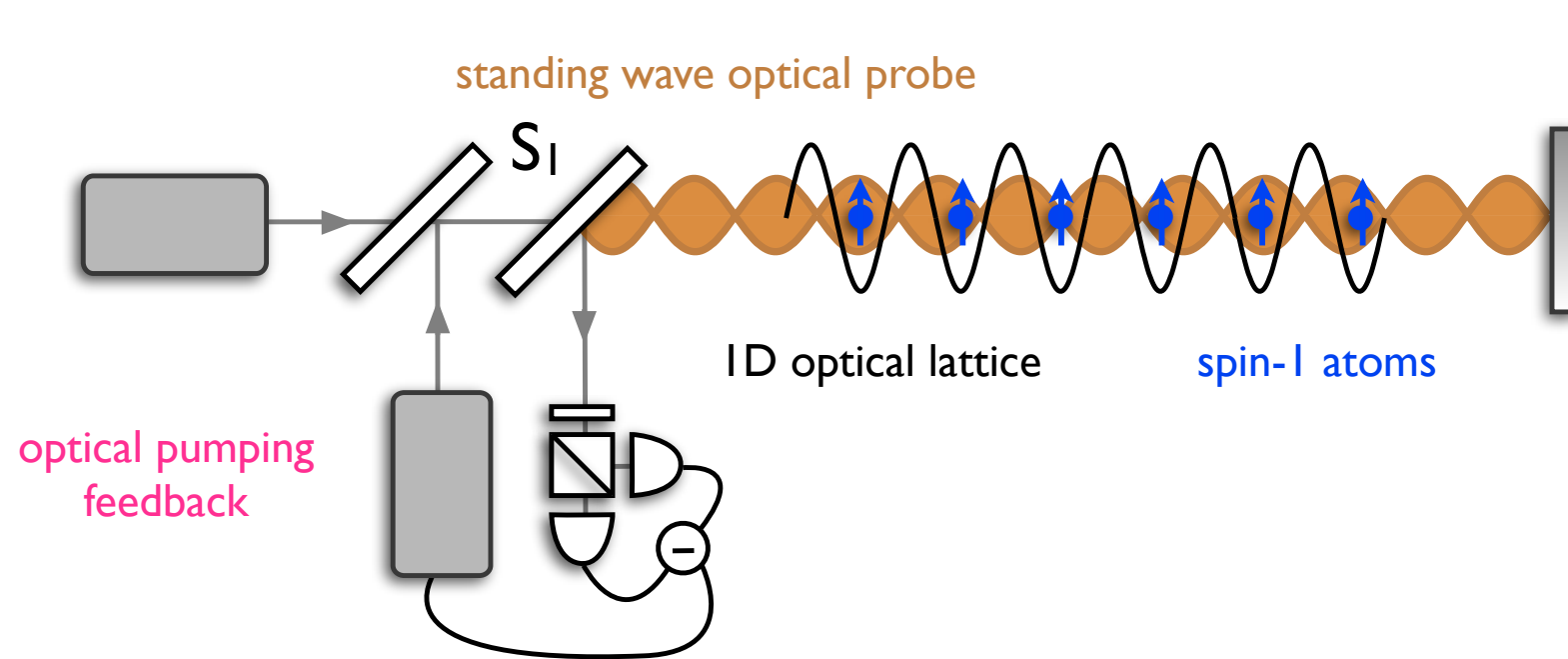


## Haldane phase

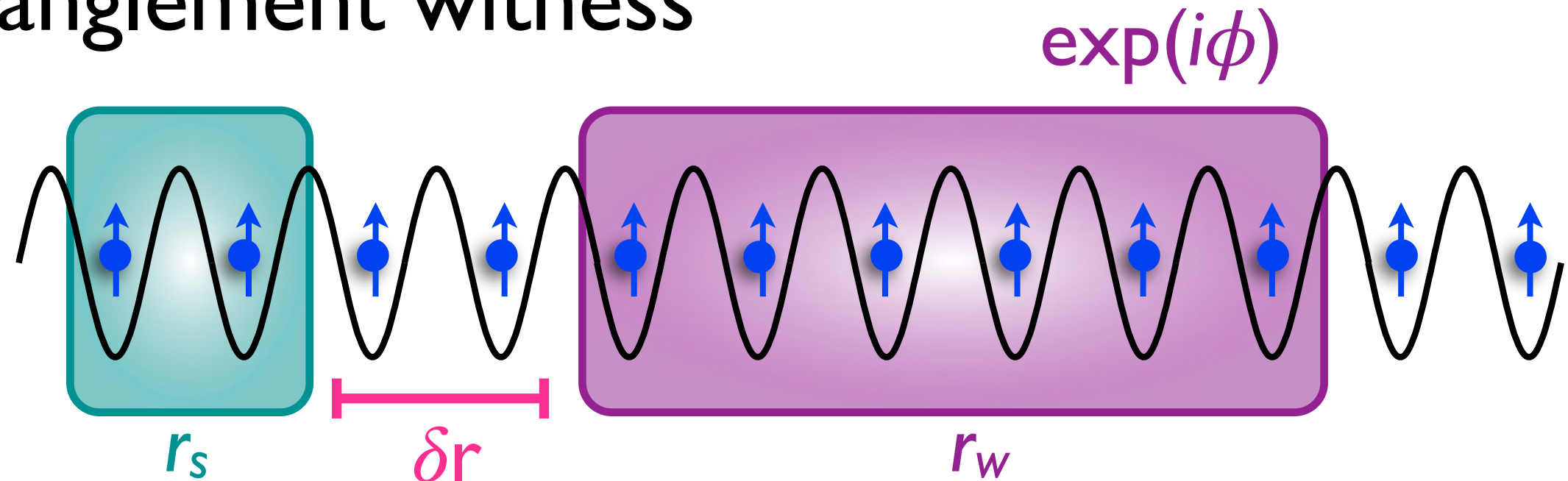


Hauke, PRA 87, 021601 (2013)





# Entanglement witness



$$W \equiv \mathcal{S}/n_a - 1 < 0$$

$$\mathcal{S} \equiv \sum_{\alpha} \mathcal{S}_{\alpha} = \sum_{\alpha} \sum_{i,j=1}^{n_s} \langle J_{\alpha,i} J_{\alpha,j} \rangle f^*(r_i) f(r_j)$$

$$f(r_i) = \begin{cases} 1 & \text{if } r_i \in r_s, \\ \exp(i\phi) & \text{if } r_i \in r_w, \\ 0 & \text{otherwise.} \end{cases}$$



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